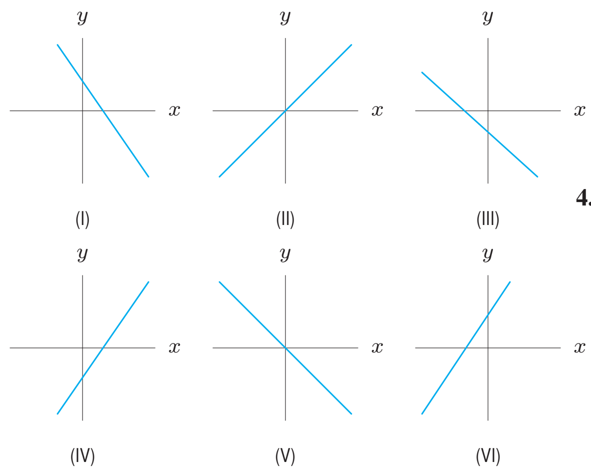
Exercises

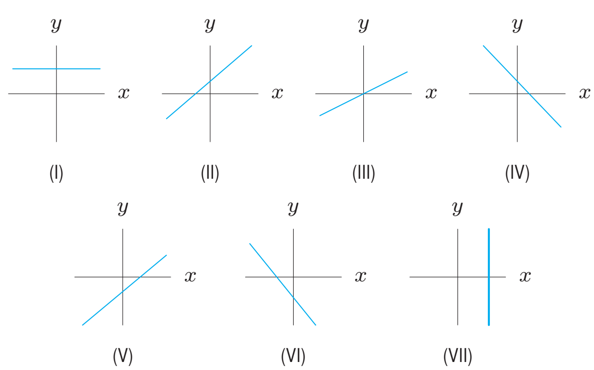
1. Without using a calculator, match the equations (a) – (f) to the graphs (I) – (VI).

(a) $y = -2.72x$ **(b)** $y = 0.01 + 0.001x$
(c) $y = 27.9 - 0.1x$ **(d)** $y = 0.1x - 27.9$
(e) $y = -5.7 - 200x$ **(f)** $y = x/3.14$



2. Without a calculator, match the equations (a)–(g) to the graphs (I)–(VII).

(a) $y = x - 5$ **(b)** $-3x + 4 = y$
(c) $5 = y$ **(d)** $y = -4x - 5$
(e) $y = x + 6$ **(f)** $y = x/2$
(g) $5 = x$



3. Figure 1.45 gives lines, A, B, C, D, and E. Without a calculator, match each line to f, g, h, u or v:

$f(x) = 20 + 2x$
$g(x) = 20 + 4x$
$h(x) = 2x - 30$
$u(x) = 60 - x$
$v(x) = 60 - 2x$

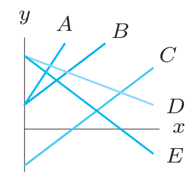


Figure 1.45

4. Without a calculator, match the following functions to the lines in Figure 1.46:

$f(x) = 5 + 2x$
$g(x) = -5 + 2x$
$h(x) = 5 + 3x$
$j(x) = 5 - 2x$
$k(x) = 5 - 3x$

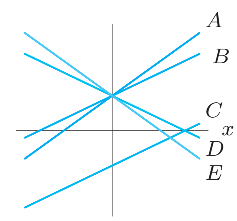


Figure 1.46

5. **(a)** By hand, graph $y = 3$ and $x = 3$.
(b) Can the equations in part (a) be written in slope-intercept form?

Are the lines in Exercises 6–11 perpendicular? Parallel? Neither?

6. $y = 5x - 7$; $y = 5x + 8$

7. $y = 4x + 3$; $y = 13 - \frac{1}{4}x$

8. $y = 2x + 3$; $\quad y = 2x - 7$

9. $y = 4x + 7$; $\quad y = \frac{1}{4}x - 2$

10. $f(q) = 12q + 7$; $g(q) = \frac{1}{12}q + 96$

11. $2y = 16 - x$; $4y = -8 - 2x$

Problems

12. Sketch a family of functions $y = -2 - ax$ for five different values of a with $a < 0$.

13. Find the equation of the line parallel to $3x + 5y = 6$ and passing through the point $(0, 6)$.

14. Find the equation of the line passing through the point $(2, 1)$ and perpendicular to the line $y = 5x - 3$.

15. Find the equations of the lines parallel to and perpendicular to the line $y + 4x = 7$, and through the point $(1, 5)$.

16. Estimate the slope of the line in Figure 1.47 and find an approximate equation for the line.

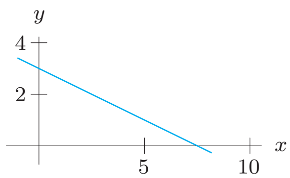


Figure 1.47

17. Line l in Figure 1.48 is parallel to the line $y = 2x + 1$. Find the coordinates of the point P.

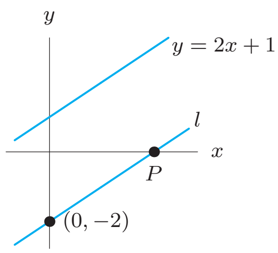


Figure 1.48

18. Find the equation of the line l_2 in Figure 1.49.

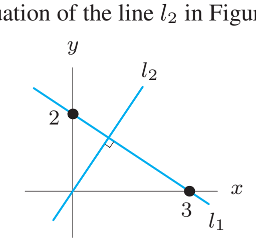


Figure 1.49

19. The cost of a Frigbox refrigerator is \$950, and it depreciates \$50 each year. The cost of an Arctic Air refrigerator is \$1200, and it depreciates \$100 per year.

(a) If a Frigbox and an Arctic Air are bought at the same time, when do the two refrigerators have equal value?

(b) If both refrigerators continue to depreciate at the same rates, what happens to the values of the refrigerators in 20 years' time? What does this mean?

20. You need to rent a car and compare the charges of three different companies. Company A charges 20 cents per mile plus \$20 per day. Company B charges 10 cents per mile plus \$35 per day. Company C charges \$70 per day with no mileage charge.

(a) Find formulas for the cost of driving cars rented from companies A, B, and C, in terms of x, the distance driven in miles in one day.

(b) Graph the costs for each company for $0 \le x \le 500$. Put all three graphs on the same set of axes.

(c) What do the slope and the vertical intercept tell you in this situation?

(d) Use the graph in part (b) to find under what circumstances company A is the cheapest. What about Company B? Company C? Explain why your results make sense.

21. Line l is given by $y = 3 - \frac{2}{3}x$ and point P has coordinates $(6, 5)$.

(a) Find the equation of the line containing P and parallel to l.

(b) Find the equation of the line containing P and perpendicular to l.

(c) Graph the equations in parts (a) and (b).

22. Assume A, B, C are constants with $A \ne 0$, $B \ne 0$. Consider the equation

$$Ax + By = C.$$

(a) Show that $y = f(x)$ is linear. State the slope and the x- and y-intercepts of $f(x)$.

(b) Graph $y = f(x)$, labeling the x- and y-intercepts in terms of A, B, and C, assuming

(i) $A > 0, B > 0, C > 0$

(ii) $A > 0, B > 0, C < 0$

(iii) $A > 0, B < 0, C > 0$

23. Fill in the missing coordinates for the points in the following figures.

(a) The triangle in Figure 1.50.

(b) The parallelogram in Figure 1.51.

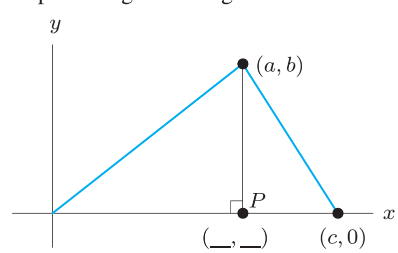


Figure 1.50

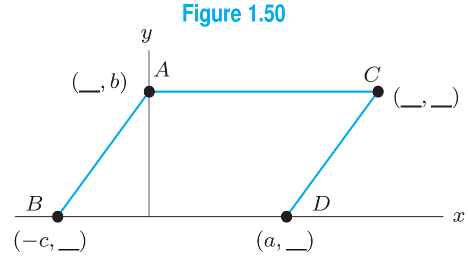


Figure 1.51

24. Using the window $-10 \leq x \leq 10, -10 \leq y \leq 10$, graph $y = x$, $y = 10x$, $y = 100x$, and $y = 1000x$.

(a) Explain what happens to the graphs of the lines as the slopes become large.
(b) Write an equation of a line that passes through the origin and is horizontal.

25. Graph $y = x + 1$, $y = x + 10$, and $y = x + 100$ in the window $-10 \leq x \leq 10, -10 \leq y \leq 10$.

(a) Explain what happens to the graph of a line, $y = b + mx$, as b becomes large.
(b) Write a linear equation whose graph cannot be seen in the window $-10 \leq x \leq 10, -10 \leq y \leq 10$ because all its y-values are less than the y-values shown.

26. The graphical interpretation of the slope is that it shows steepness. Using a calculator or a computer, graph the function $y = 2x - 3$ in the following windows:

(a) $-10 \leq x \leq 10$ by $-10 \leq y \leq 10$
(b) $-10 \leq x \leq 10$ by $-100 \leq y \leq 100$
(c) $-10 \leq x \leq 10$ by $-1000 \leq y \leq 1000$
(d) Write a sentence about how steepness is related to the window being used.

In Problems 27–28, what is true about the constant β in the following linear equation if its graph has the given property?

$$y = \frac{x}{\beta - 3} + \frac{1}{6 - \beta}.$$

27. Positive slope, positive y-intercept.

28. Perpendicular to the line $y = (\beta - 7)x - 3$.

29. A circle of radius 2 is centered at the origin and goes through the point $(-1, \sqrt{3})$.

(a) Find an equation for the line through the origin and the point $(-1, \sqrt{3})$.
(b) Find an equation for the tangent line to the circle at $(-1, \sqrt{3})$. [Hint: A tangent line is perpendicular to the radius at the point of tangency.]

30. Find an equation for the altitude through point A of the triangle ABC, where A is $(-4, 5)$, B is $(-3, 2)$, and C is $(9, 8)$. [Hint: The altitude of a triangle is perpendicular to the base.]

31. Fill in the missing coordinates in Figure 1.52. Write an equation for the line connecting the two points. Check your answer by solving the system of two equations.

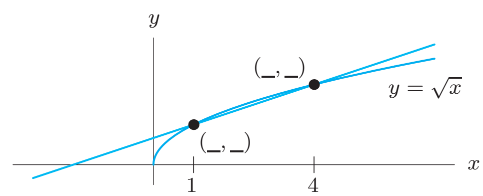


Figure 1.52

32. Two lines are given by $y = b_1 + m_1 x$ and $y = b_2 + m_2 x$, where b_1, b_2, m_1, and m_2 are constants.

(a) What conditions are imposed on b_1, b_2, m_1, and m_2 if the two lines have no points in common?
(b) What conditions are imposed on b_1, b_2, m_1, and m_2 if the two lines have all points in common?
(c) What conditions are imposed on b_1, b_2, m_1, and m_2 if the two lines have exactly one point in common?
(d) What conditions are imposed on b_1, b_2, m_1, and m_2 if the two lines have exactly two points in common?

1.6 FITTING LINEAR FUNCTIONS TO DATA

When real data are collected in the laboratory or the field, they are often subject to experimental error. Even if there is an underlying linear relationship between two quantities, real data may not fit this relationship perfectly. However, even if a data set does not perfectly conform to a linear function, we may still be able to use a linear function to help us analyze the data.

Laboratory Data: The Viscosity of Motor Oil

The viscosity of a liquid, or its resistance to flow, depends on the liquid's temperature. Pancake syrup is a familiar example: straight from the refrigerator, it pours very slowly. When warmed on the stove, its viscosity decreases and it becomes quite runny.

The viscosity of motor oil is a measure of its effectiveness as a lubricant in the engine of a car. Thus, the effect of engine temperature is an important determinant of motor-oil performance. Table 1.33 gives the viscosity, v, of motor oil as measured in the lab at different temperatures, T.