10.1 – Defining Convergent and Divergent Infinite Series Name ______________________________

In the following **sequences**, determine the convergence or divergence with the given nth term. Be sure to properly justify your conclusion.

1.) $\{a_n\} = \left\{(-1)^n \left(\frac{n}{n+1}\right)\right\}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left[(-1)^n \frac{n}{n+1}\right] = \pm 1 \Rightarrow \text{diverges}$$

2.) $\{a_n\} = \left\{\frac{-2\sqrt{n}}{3\sqrt{n}+1}\right\}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left[\frac{-2\sqrt{n}}{3\sqrt{n}+1}\right] = \lim_{n\to\infty}\left[\frac{-2\sqrt{n}}{3\sqrt{n}}\right] = -\frac{2}{3}$$

converges

3.) $\{a_n\} = \left\{\frac{3n^2-3n+8}{4n^2+1}\right\}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left[\frac{3n^2-3n+8}{4n^2+1}\right] = \frac{3}{4} \Rightarrow \text{converges}$$

4.) $\{a_n\} = \{n \pm (-1)^n\}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left[n \pm (-1)^n\right] = \infty \pm 1 = \infty \Rightarrow \text{diverges}$$

Find the sequence of partial sums, S_1, S_2, S_3, S_4.

5.) $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots$

$S_1 = 1$ $\quad S_2 = 1 + \frac{1}{4} = \frac{5}{4}$ $\quad S_3 = \frac{5}{4} + \frac{1}{9} = \frac{49}{36}$

$S_4 = \frac{49}{36} + \frac{1}{16} = \frac{205}{144}$

6.) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!}$

$S_1 = 1$ $\quad S_2 = 1 - \frac{1}{2} = \frac{1}{2}$ $\quad S_3 = \frac{1}{2} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

$S_4 = \frac{2}{3} - \frac{1}{24} = \frac{15}{24} = \frac{5}{8}$

10.2 & 10.3 – Geometric Series Test and nth term Test

In the following exercises, determine the convergence or divergence of the series. Be sure to properly justify your conclusion.

7.) $\displaystyle\sum_{n=1}^{\infty} \frac{n+5}{5n+2}$

$$\lim_{n\to\infty} \frac{n+5}{5n+2} = \lim_{n\to\infty} \frac{n}{5n} = \frac{1}{5}$$

diverges by nth Term Test

8.) $\displaystyle\sum_{n=1}^{\infty} \frac{n+3}{2n-1}$

$$\lim_{n\to\infty} \frac{n+3}{2n-1} = \lim_{n\to\infty} \frac{n}{2n} = \frac{1}{2}$$

diverges by nth Term Test

9.) $\displaystyle\sum_{n=1}^{\infty} (\sin 1)^n$

Geometric with $r = \sin(1) < 1$

converges by the Geometric Test

10.) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

$\displaystyle\lim_{n\to\infty} \frac{2^n}{n^2} = \infty$ because the exponential dominates

diverges by nth Term Test

11.) $\displaystyle\sum_{n=0}^{\infty} \frac{9}{3^n}$

Geometric with $r = \frac{1}{3} < 1$

converges by the Geometric Test

12.) $\displaystyle\sum_{n=2}^{\infty} \frac{n}{10 \ln n}$

$$\lim_{n\to\infty} \frac{n}{10\ln n} = \begin{cases} \lim_{n\to\infty} n = \infty \\ \lim_{n\to\infty} 10\ln n = \infty \end{cases} \Rightarrow \text{indeterminate } \frac{\infty}{\infty}$$

$$\lim_{n\to\infty} \frac{n}{10\ln n} = \underbrace{\lim_{n\to\infty} \frac{1}{10/n}}_{\text{l'Hospital's Rule}} = \lim_{n\to\infty} \frac{n}{10} = \infty$$

diverges by the nth Term Test