## nth-Term Test for Divergence

This topic begins with a couple theorems.

## THEOREM 10.3A: LIMIT OF THE $n$ TH TERM OF A CONVERGENT SERIES

$$
\text { If } \sum_{n=1}^{\infty} a_{n} \text { converges, then } \lim _{n \rightarrow \infty} a_{n}=0
$$

It seems rather intuitive that a series cannot possibly add to a finite sum unless the terms themselves approached 0 . What is important from the above theorem is that its contrapositive is true as well.

## THEOREM 10.3B: $n$ TH TERM TEST FOR DIVERGENCE

$$
\text { If } \lim _{n \rightarrow \infty} a_{n} \neq 0 \text {, then } \sum_{n=1}^{\infty} a_{n} \text { diverges. }
$$

## Example 1: Using the nth-Term Test for Divergence

Determine the convergence or divergence of each series.
a.) $\sum_{n=0}^{\infty} \frac{n}{n+1}$
b.) $\sum_{n=1}^{\infty} \frac{1+3^{n}}{2^{n}}$
c.) $\sum_{n=1}^{\infty} \frac{1}{n}$
d.) $\sum_{n=1}^{\infty} \frac{n^{3}}{n!}$

1. It is very important to note that if

$$
\lim _{n \rightarrow \infty} a_{n}=0
$$

the series does not necessarily converge.
This just means the nth Term Test is inconclusive.
2. You will start to see more factorials in your series expressions. They can be somewhat tricky to deal with as they "do not play nicely" with the calculus that we know. Thinking of the logical impact they have on the series expression is a good strategy.

