Series and Convergence 10.8 HWK

Name ______________

Use the Ratio Test to determine the convergence or divergence of each of the following series.

1.) $\displaystyle\sum_{n=0}^{\infty}\frac{n!}{4^n}$

$$\lim_{n\to\infty}\left|\frac{(n+1)!}{4^{n+1}}\cdot\frac{4^n}{n!}\right|$$

$$=\lim_{n\to\infty}\left|\frac{n+1}{4}\right|=\infty$$

Diverges

2.) $\displaystyle\sum_{n=0}^{\infty}\frac{4^n}{n!}$

$$\lim_{n\to\infty}\left|\frac{4^{n+1}}{(n+1)!}\cdot\frac{n!}{4^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{4}{n+1}\right|=0<1$$

Converges

3.) $\displaystyle\sum_{n=1}^{\infty}n\left(\frac{3}{4}\right)^n$

$$\lim_{n\to\infty}\left|\frac{n+1}{1}\cdot\left(\frac{3}{4}\right)^{n+1}\cdot\frac{1}{n}\cdot\left(\frac{4}{3}\right)^n\right|$$

$$\lim_{n\to\infty}\left|\frac{3}{4}\cdot\frac{n+1}{n}\right|=\frac{3}{4}<1$$

Converges

4.) $\displaystyle\sum_{n=1}^{\infty}\frac{3^n}{n^3}$

$$\lim_{n\to\infty}\left|\frac{3^{n+1}}{(n+1)^3}\cdot\frac{n^3}{3^n}\right|$$

$$\lim_{n\to\infty}\left|3\cdot\frac{n^3}{(n+1)^3}\right|=3>1$$

Diverges

5.) $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{(n+1)!}$

$$\lim_{n\to\infty}\left|\frac{2^{n+1}}{(n+2)!}\cdot\frac{(n+1)!}{2^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{2}{n+2}\right|=0<1$$

Converges

6.) $\displaystyle\sum_{n=1}^{\infty}\frac{5^n}{3^n+1}$

$$\lim_{n\to\infty}\left|\frac{5^{n+1}}{3^{n+1}+1}\cdot\frac{3^n+1}{5^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{5}{1}\cdot\frac{3^n+1}{3^{n+1}+1}\right|=\frac{5}{3}>1$$

Diverges

Test for convergence or divergence of the following using any test that we have discussed in this unit. Identify the test and show your work.

7.) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}4}{n}$

$\lim_{n\to\infty} a_n = 0$; $\frac{4}{1} > \frac{4}{2} > \frac{4}{3} \cdots$

Converges by AST

8.) $\underbrace{\sum_{n=1}^{\infty} \frac{6}{n}}_{b_n}$

$\boxed{\text{or}}$ $6 \sum \frac{1}{n}$ = Harmonic

$\underbrace{\sum \frac{1}{n}}_{a_n}$ Diverges

Diverges by Direct Comparisin Test

9.) $\sum_{n=1}^{\infty} \frac{n}{2^n+5}$

$\lim_{n\to\infty} \frac{n}{2^n+5} = 0$

$\sum \frac{1}{2^n}$ converges

$\lim_{n\to\infty} \left(\frac{n}{2^n+5} \cdot \frac{2^n}{1}\right) = \infty$ Inconclusive

$\lim_{n\to\infty} \left|\frac{n+1}{2^{n+1}+5} \cdot \frac{2^n+5}{n}\right| = \frac{1}{2} < 1$ Converges by Ratio Test

10.) $\sum_{n=1}^{\infty} \frac{7}{2\sqrt{n^3}}$

$\frac{7}{2} \sum \frac{1}{n^{3/2}}$ Converges

Converges by p-series test

11.) $\sum_{n=1}^{\infty} \frac{n5^n}{n!}$

$\lim_{n\to\infty} \left|\frac{(n+1)5^{n+1}}{(n+1)!} \cdot \frac{n!}{n5^n}\right|$

$\lim_{n\to\infty} \left|\frac{n+1}{n} \cdot \frac{5}{n+1}\right|$

$\lim_{n\to\infty} \left|\frac{5}{n}\right| = 0 < 1$

Converges by Ratio Test

12.) $\underbrace{\sum_{n=1}^{\infty} \frac{\ln n}{n^2}}_{a_n}$

$\underbrace{\sum \frac{1}{n^2}}_{b_n}$ Converges

Converges by Comparison Test