Series and Convergence 10.9 & 10.10 HWK Name odds

Determine whether the series converges absolutely or conditionally, or diverges.

1. $\sum_{n=1}^{\infty}\frac{(-1)^n}{2^n}$ Converges by AST

$\sum \frac{1}{2^n}$ Converges by GST

Absolute Convergence

2. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n}}$ Converges by AST

$\sum \frac{1}{n^{1/2}}$ Diverges by p-test

Conditional

3. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}n^2}{(n+1)^2}$ Diverges by n^{th} term test

4. $\sum_{n=0}^{\infty}\frac{(-1)^n}{\sqrt{n+4}}$ Converges by AST

Conditional

$\sum \frac{1}{\sqrt{n+4}}$ compare $\sum \frac{1}{n^{1/2}}$ diverges

a_n b_n

$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n+4}}\cdot\frac{\sqrt{n}}{1}\right)=1$ Diverges by Limit Comparison

5. $\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n^2}$ Converges by AST

$\sum\left|\frac{\cos n\pi}{n^2}\right| = \sum\frac{1}{n^2}$

$|\cos\pi| = 1$; $|\cos 2\pi| = 1$; ...

Converges by p-series test

Absolute

Approximate the sum of each series by using the first five terms. Then find the error. State the interval of convergence. You may use your calculator.

6. $\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)!}$

$S \approx S_5 = \sum_{n=0}^{4} \frac{(-1)^n}{(n+1)!} \approx 0.6\overline{33}$ overestimate (max)

Error $= |a_6| = \left|\frac{(-1)^5}{(5+1)!}\right| = \frac{1}{720}$ (6th term)

$a_6 < 0$

$.63194 \leq S < 0.6\overline{33}$

7. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$

$S \approx S_5 = \sum_{n=0}^{4} \frac{(-1)^n}{2n+1} \approx .8349$ overestimate

$a_6 < 0$

Error $= |a_6| = \left|\frac{(-1)^5}{10+1}\right| = \frac{1}{11}$; $a_6 < 0$

$.7439 \leq S < .8349$

8. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

$S \approx S_5 = \sum_{n=1}^{5} \frac{(-1)^{n+1}}{n^2} \approx .8386$ overestimate

$a_6 < 0$

Error $= |a_6| = \left|\frac{(-1)^7}{36}\right| = \frac{1}{36}$

$.8108 \leq S < .8386$

Determine the number of terms required to approximate the sum of the series with an error less than 0.001.

9. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$

S_{10}

$\frac{1}{(n+1)^3} < \frac{1}{1000}$

$n+1 > \sqrt[3]{1000}$ $\Rightarrow n > 10-1$, $n > 9$

OR

$\frac{1}{n^3} < 0.001$

$n^3 > 1000$

11th term is error $< .001$

10. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n^3-1}$

S_7

$\frac{1}{2(n+1)^3-1} < \frac{1}{1000}$

$1000 < 2(n+1)^3 - 1$

$\frac{999}{2} < (n+1)^3$ $\Rightarrow n > 6.934$

$\frac{1}{2n^3-1} < 0.001$

$n > 7.939$

8th term is error $< .001$

11. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

S_6

$\frac{1}{(n+1)!} < \frac{1}{1000}$

How do you solve?

$\frac{1}{n!} < 0.001$

$n! > 1000$

$6! = 720$ $7! = 5040$

7th term is error $< .001$