①

$$f(x) = \sum_{n=1}^{\infty} \frac{x^{2n}}{n!} \quad ; \quad f'(x) = \sum_{n=1}^{\infty} 2n \cdot \frac{x^{2n-1}}{n!}$$

②

$C = 4$

Given: (Fact)

$1 < x \leq 7$ or $-1 < x < 9$

③

$$\lim_{n\to\infty} \left| \frac{(x+2)^{n+1}}{2^{n+1}} \cdot \frac{2^n}{(x+2)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{1}{2} \cdot (x+2) \right| = \left| \frac{1}{2}(x+2) \right|$$

$$-1 < \frac{1}{2}(x+2) < 1$$

$$-2 < x+2 < 2$$

$$-4 < x < 0$$

<u>Check -4:</u>

$$\sum_{n=0}^{\infty} \frac{(-4+2)^n}{2^n} = \sum_{n=0}^{\infty} \frac{(-2)^n}{2^n} = \sum_{n=0}^{\infty} (-1)^n \text{ diverges}$$

(4)

$$\sum_{n=1}^{\infty} \frac{(x+4)^n}{n \cdot 5^{n+1}}$$

$$\lim_{n\to\infty} \left| \frac{(x+4)^{n+1}}{(n+1)\cdot 5^{n+2}} \cdot \frac{n \cdot 5^{n+1}}{(x+4)^n} \right|$$

$$= \lim_{n\to\infty} \left| (x+4) \cdot \frac{n}{n+1} \cdot \frac{1}{5} \right| = \left| \frac{x+4}{5} \right| \cdot 1$$

$$-1 < \frac{x+4}{5} < 1$$

$$-5 < x+4 < 5$$

$$-9 < x < 1$$

No need to check endpoints based on possible answers listed

(5) $f'(x) = x^2 - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!}$

$$\int f'(x) = \frac{x^3}{3} - \frac{x^7}{7 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!} + C$$

(6) $\dfrac{2x}{1-(-x^2)}$ $a_1 = 2x$

$r = |-x^2|$

$|-x^2| < 1$ Geometric center at 0

(7) $f(x) = 2 + 3x + 1x^2 + \frac{1}{3}x^3 + \ldots$

$f(0) = 2$

$f'(0) = 3$

$\dfrac{f''(0)}{2!} = 1 \Rightarrow f''(0) = 2$

$g(x) = e^{f(x)}$

$g'(x) = e^{f(x)} \cdot f'(x)$

$g''(x) = e^{f(x)} \cdot f''(x) + f'(x) \cdot e^{f(x)} \cdot f'(x)$

$g''(0) = e^{f(0)} \cdot f''(0) + f'(0) \cdot e^{f(0)} \cdot f'(0)$

$= e^2 \cdot 2 + 3^2 \cdot e^2$

$= 11e^2$

Find coefficient of x^2

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$$x \cdot \cos(x^2) = x \cdot \left[1 - \frac{(x^2)^2}{2!} + \frac{(x^2)^4}{4!} - \frac{(x^2)^6}{6!} + \dots\right]$$

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$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x \qquad \sum_{n=0}^{\infty} \frac{(x^2)^n}{n!} = e^{x^2}$$

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$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots + \frac{x^n}{n!}$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \dots + \frac{(-x)^n}{n!}$$