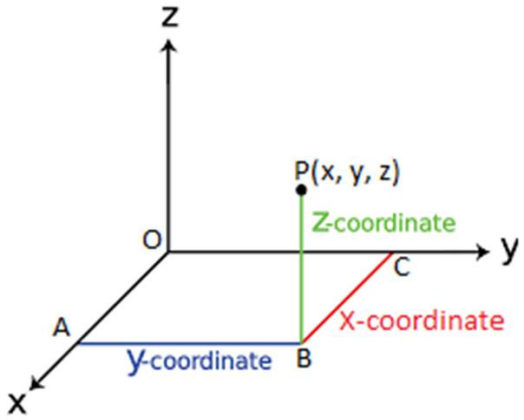
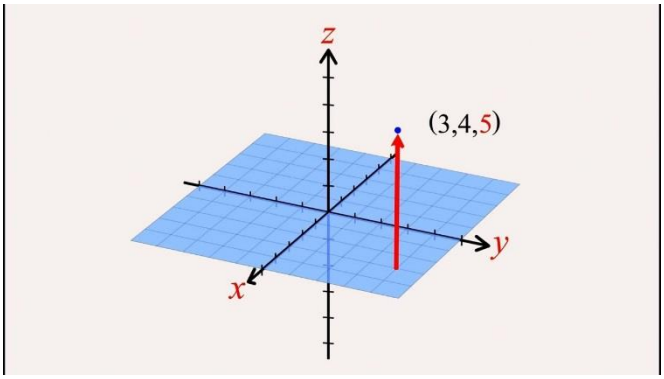
Notes 11. 1 The Three – Dimensional Coordinate System

To identify a point in space, you must introduce a third dimension. You can construct a **three-dimensional coordinate system** by passing a *z*-axis perpendicular to both the *x*- and *y*-axes at the origin.





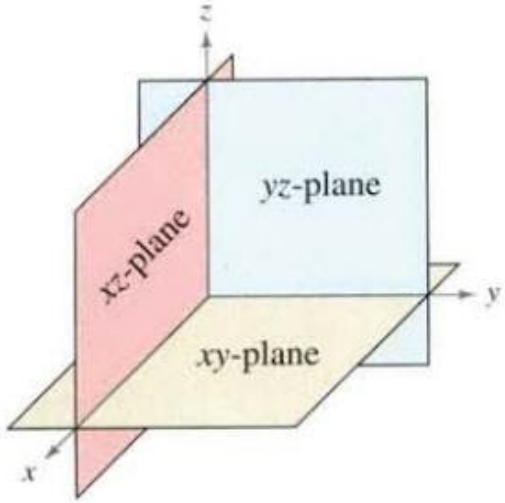
Ex. 1 Plot the points

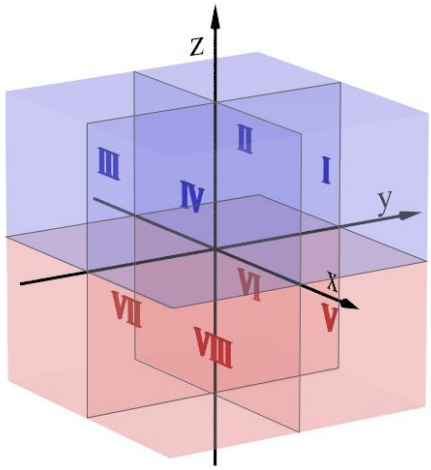
a.) $(2, -3, 4)$

b.) $(-2, 6, 2)$

Taken as pairs, the axes determine three **coordinate planes**: the ***xy***-plan, the ***xz***-plane, and the ***yz***-plane. These three coordinate planes separate the three-dimensional coordinate system into eight **octants.**

In this text, we work exclusively with right-handed systems. In a right-handed system, Octants II, III, IV are found by rotating counterclockwise around the positive *z*-axis. Octant V is vertically below Octant I. Octants VI, VII, VII are then found by rotating counterclockwise around the negative *z*-axis.





Below is a table that can help figure out the signs of the octants or the octant in which points lie.

Octants								
Co-ordinates	I	II	III	IV	V	VI	VII	VIII
x	+	−	−	+	+	−	−	+
y	+	+	−	−	+	+	−	−
z	+	+	+	+	−	−	−	−

Ex. 2 Which octants do the points in Ex. 1 lie?

Distance Formula in Space

The distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2) given by the **Distance Formula in Space** is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Midpoint Formula in Space

The midpoint of the line segment joining the points (x_1, y_1, z_1) and (x_2, y_2, z_2) given by the **Midpoint Formula in Space** is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right).$$

Ex. 3 $(5, -2, 3)$ and $(0, 4, 4)$

a.) Plot the points

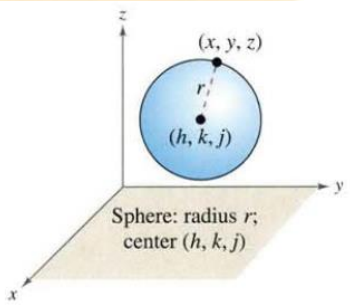
b.) Find the distance between the points

c.) Find the midpoint

Standard Equation of a Sphere

The **standard equation of a sphere** whose center is (h, k, j) and whose radius is r is

$$(x - h)^2 + (y - k)^2 + (z - j)^2 = r^2.$$



Ex. 4 Find the center and radius of the following sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z + 8 = 0$

Finding the **trace** of a surface consists of all points that are in common to both the surface AND the plane.

Ex. 5 Show the work to find the *xy*-trace in the figure to the right.