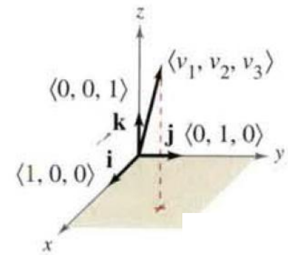
Notes 11. 2 Vectors in Space

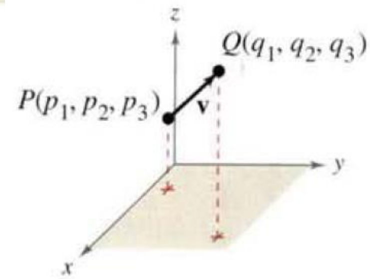


Component form: $\boldsymbol{v} = \langle v_{1\,,}v_2, v_3\rangle$ *(terminal point)*

Zero vector: $\mathbf{0} = \langle 0,0,0\rangle$ (*initial point)*

Unit vectors: $\boldsymbol{i} = \langle 1,0,0\rangle$ $\boldsymbol{j} = \langle 0,1,0\rangle$ $\boldsymbol{k} = \langle 0,0,1\rangle$

Unit vector form: $\boldsymbol{v} = v_1\boldsymbol{i} + v_2\boldsymbol{j} + v_3\boldsymbol{k}$



If ***v*** is represented by the directed line segment from P to Q as shown in the figure, the **component form** of ***v*** is produced by subtracting the coordinate of the initial point from the coordinates of the terminal point.

Vectors in Space

1. Two vectors are equal if and only if their corresponding components are equal.
2. The magnitude (or length) of $\mathbf{u} = \langle u_1, u_2, u_3\rangle$ is $\|\mathbf{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2}$.
3. A unit vector $\mathbf{u}$ in the direction of $\mathbf{v}$ is $\mathbf{u} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|}$, $\mathbf{v} \neq \mathbf{0}$. (**unit** vector has a length of **1**)
4. The sum of $\mathbf{u} = \langle u_1, u_2, u_3\rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3\rangle$ is

   $\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3\rangle.$ Vector addition
5. The scalar multiple of the real number c and $\mathbf{u} = \langle u_1, u_2, u_3\rangle$ is

   $c\mathbf{u} = \langle cu_1, cu_2, cu_3\rangle.$ Scalar multiplication
6. The dot product of $\mathbf{u} = \langle u_1, u_2, u_3\rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3\rangle$ is

   $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3.$ Dot product

Ex. 1

a.) Find the component form of vector ***v*** having an initial point $\langle 2,4,1\rangle$ and a terminal point $\langle 3,5,2\rangle$.

b.) Find the magnitude of vector ***v*** having an initial point $\langle 2,4,1\rangle$ and a terminal point $\langle 3,5,2\rangle$.

c.) Find the unit vector in the direction of ***v.***

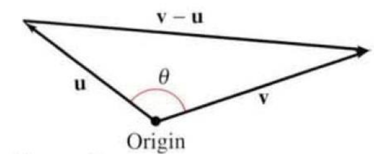
Ex. 2 Find the dot product. $u = \langle -1,3,5 \rangle$ $v = \langle 0,4,-2 \rangle$

The angle between two non-zero vectors is angle $\theta, 0 \le \theta \le \pi$. This angle can be found by using the **dot product**. If the dot product equals 0 then the angle between the two vectors is 90° and such vectors are **orthogonal**. **Orthogonal** vectors are perpendicular.

Angle Between Two Vectors

If θ is the angle between two nonzero vectors **u** and **v**, then

$$\cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \, \|\mathbf{v}\|}.$$



Ex. 3. Find the angle between the vectors. $u = \langle 1,0,2 \rangle$ $v = \langle 3,1,0 \rangle$

Parallel Vectors

Recall from the definition of scalar multiplication that positive scalar multiples of a nonzero vector **v** have the same direction as **v**, whereas negative multiples have the direction opposite that of **v**. In general, two nonzero vectors **u** and **v** are **parallel** if there is some scalar c such that $\mathbf{u} = c\mathbf{v}$. For example, in Figure 11.13, the vectors **u**, **v**, and **w** are parallel because $\mathbf{u} = 2\mathbf{v}$ and $\mathbf{w} = -\mathbf{v}$.

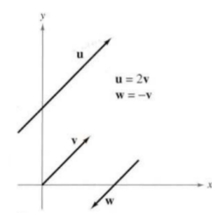


Figure 11.13

Ex. 4 Vector ***w*** has initial point $(1,-2,0)$ and terminal point $(3,2,1)$. Find the component form of vector ***w.*** Which of the following is parallel to **w?**

a.) $u = \langle 4,8,2 \rangle$

b.) $v = \langle 4,8,4 \rangle$

Collinear points- Points ***P, Q,*** and ***R*** are collinear (on the same line) if $\overrightarrow{PQ} \mathbin{//} \overrightarrow{PR}$ (same slope)

Ex. 5 Determine if the following points are collinear. $P: (2,-1,4)$; $Q: (5,4,6)$; $R: (-4,-11,0)$