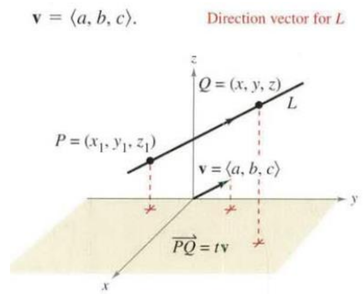
Notes 11.4 Lines and Planes in Space

- Usually, slope is used to determine an equation of a line
- In space, it is more convenient to use vectors to determine the equation of a line
- in the figure to the right, vector ***v*** is the **direction vector** for the line L, and a,b, and c are the **direction numbers**
- One way of describing line *L,* is to say that is consists of all points $Q = (x, y, z)$ for which vector $\overrightarrow{PQ}$ is parallel to ***v.***
- Therefore, $\overrightarrow{PQ}$ is a scalar multiple of ***v.***
- $\overrightarrow{PQ} = \langle at, bt, ct \rangle = t \cdot \boldsymbol{v}$



Parametric Equations of a Line in Space

A line L parallel to the **vector** $\mathbf{v} = \langle a, b, c \rangle$ and passing through the point $P = (x_1, y_1, z_1)$ is represented by the parametric equations

$$x = x_1 + at \qquad y = y_1 + bt \qquad z = z_1 + ct.$$

By equating corresponding components, you can obtain the **parametric equations of the line in space.**

If $\boldsymbol{v} = \langle a, b, c \rangle$ is the direction vector and if *a, b, and c* are all nonzero, you can eliminate the parameter *t* to obtain the **symmetric equations** of the line which is an *alternate* way to express a distinct line in three space. It is considered more compact.

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad \text{Symmetric equations}$$

Ex. 1 Find a set of (a) parametric equations and (b) symmetric equations for the line that passes through $(-3,0,2)$ and parallel to $\vec{v} = \langle 1,6,3 \rangle$

a.)

b.)

Ex. 2

a.) Find a set of (a) parametric equations and (b) symmetric equations for the line that passes through $(0,4,3)$ and $(-1,2,5)$

Hint: Find the component form of $\overrightarrow{PQ} = \langle x - x_1, y - y_1, z - z_1 \rangle$ if given two points. This would be the direction vector you use with point ***P.***

Ex. 3 Determine which of the points lie on the line that passes through the point$(-2,3,1)$ and is parallel to the vector $\boldsymbol{v} = 4i - k$

(a) $(2,3,0)$

(b) $(-6,3,2)$

(c) $(2,1,0)$

(d) $(6,3,-2)$