AP CALCULUS BC	
Topic: 2.2	**Defining the Derivative of a Function and Using Derivative Notation**

Recall from Topic 2.1, $f'(c)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$. This limit, which calculates the derivative of the function *f* when $x=c$, can be expressed a little differently.
What if we wanted a general derivative that can be computed for any value of *x* (and not just at *c*)?

ACTIVITY. Consider the limit: $\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$. Let $x=c+h$. Rewrite, and simplify, the limit above using the new expression for *x*.

<u>Alternate Form of the Derivative</u>
Another way to think of the derivative of the function *f* when $x=c$ is

$$f'(c)=\lim_{h\to 0}\frac{f(c+h)-f(c)}{h}$$

What is powerful about the above definition of the derivative is that if we don't use the value *c*, but instead use the variable *x*, we have an expression that can be used to find a general derivative at any point.

<u>DEFINITION: The Derivative Function, *f'*</u>
The **derivative** function *f'* of a function *f* is

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

THINK ABOUT IT.
Sometimes the definition to the left is written as

$$f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

What do you think the meaning of Δx is?

<u>Note</u>: The word derivative is a noun. Often directions may ask you to find the derivative of a function as "*differentiate* the function." Here, differentiate is a verb.

<u>Example 1</u>: Differentiate $f(x)=\sqrt{x}$ and determine the domain of $f'(x)$.

<u>Example 2:</u> **Identifying Parts of the Definition of a Derivative**

The limits below represent $f'(c)$ for a function f and a number c. Find f and c.

a. $\lim\limits_{h \to 0} \dfrac{\sqrt{(9+h)}-3}{h}$

b. $\lim\limits_{\Delta x \to 0} \dfrac{\left[2+(-3+\Delta x)^3\right]-(-25)}{\Delta x}$

c. $\lim\limits_{x \to \frac{\pi}{3}} \dfrac{\cos x - \frac{1}{2}}{x + \frac{\pi}{3}}$