Pre-Calculus Worksheet

Name: Key

Section 2.5 - Zeros of Polynomial Functions

I. Determine the EXACT VALUES of the zeros of each polynomial. Use your calculator to get started.

1. $f(x)=13x^3-76x^2+92x+16$

from calc: 2 and 4

2	13	−76	92	16
		26	−100	−16
	13	−50	−8	0

4	13	−50	−8
		52	8
	13	2	0

$13x+2=0$

$13x=-2 \longrightarrow x=-\frac{2}{13}$

$\boxed{\left\{-\frac{2}{13}, 2, 4\right\}}$

2. $f(x)=7x^3+25x^2+16x-12$

from calc: −2 (double)

−2	7	25	16	−12
		−14	−22	12
	7	11	−6	0

−2	7	11	−6
		−14	6
	7	−3	0

$7x-3=0$

$7x=3 \longrightarrow x=\frac{3}{7}$

$\boxed{\left\{-2, \frac{3}{7}\right\}}$

double root

3. $f(x)=x^4+4x^3-17x^2-48x+60$

from calc: 1, −5

1	1	4	−17	−48	60
		1	5	−12	−60
	1	5	−12	−60	0

−5	1	5	−12	−60
		−5	0	60
	1	0	−12	0

$x^2-12=0$

$x^2=12$

$x=\pm 2\sqrt{3}$

$\boxed{\{-5, 1, \pm 2\sqrt{3}\}}$

4. $f(x)=2x^5+9x^4+6x^3-20x^2-24x$

$f(x)=\underset{0}{x}(\underbrace{2x^4+9x^3+6x^2-20x-24}_{\text{from calc: } -2 \text{ (triple)}})$

−2	2	9	6	−20	−24
		−4	−10	8	24
	2	5	−4	−12	0

−2	2	5	−4	−12
		−4	−2	12
	2	1	−6	0

−2	2	1	−6
		−4	6
	2	−3	0

$2x-3=0$

$2x=3$

$x=\frac{3}{2}$

$\boxed{\left\{0, -2, \frac{3}{2}\right\}}$

triple root

5. $f(x)=x^5+3x^4+21x^3+75x^2-100x$

$f(x)=\underset{0}{x}(\underbrace{x^4+3x^3+21x^2+75x-100}_{\text{from calc: } -4,\ 1})$

−4	1	3	21	75	−100
		−4	4	−100	100
	1	−1	25	−25	0

1	1	−1	25	−25
		1	0	25
	1	0	25	0

$x^2+25=0$

$x^2=-25$

$x=\pm 5i$

$\boxed{\{0, -4, 1, \pm 5i\}}$

6. $f(x)=3x^5-x^4-13x^3+7x^2+14x-10$

from calc: 1 (double)

1	3	−1	−13	7	14	−10
		3	2	−11	−4	10
	3	2	−11	−4	10	0

1	3	2	−11	−4	10
		3	5	−6	−10
	3	5	−6	−10	0

$3x^3+5x^2-6x-10=0$

$x^2(3x+5)-2(3x+5)=0$

$(3x+5)(x^2-2)=0$

$x=-\frac{5}{3}, \pm\sqrt{2}$

$\boxed{\left\{1, -\frac{5}{3}, \pm\sqrt{2}\right\}}$

double root

II. Determine the remaining zeros given one of the zeros. Explain how you arrived at your answer.

7. $f(x) = 2x^4 - 5x^3 + 8x^2 - 50x - 120$; $i\sqrt{7}$

If $i\sqrt{7}$ is a zero, so is $-i\sqrt{7}$.

I have 2 more zeros to find.

I graphed $f(x)$ and found that 4 was a zero. so was $-\frac{3}{2}$.

(on calculator)

$\boxed{\left\{\pm i\sqrt{7},\ 4,\ -\frac{3}{2}\right\}}$

8. $f(x) = x^5 - 15x^4 + 73x^3 - 139x^2 + 114x - 34$; $6 - \sqrt{2}$

If $6-\sqrt{2}$ is a zero, so is $6+\sqrt{2}$.

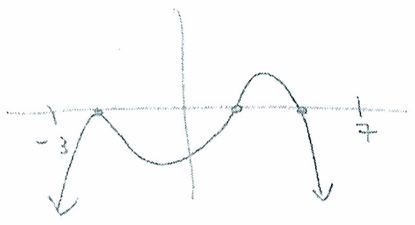
I have 3 more zeros to find.

When I graphed $f(x)$ on the calculator, I found that 1 is a triple root (bends)

$\boxed{\{6 \pm \sqrt{2},\ 1\}}$

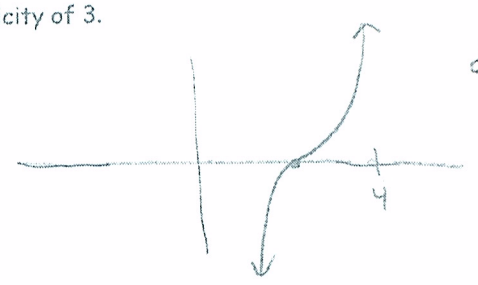
III. Sketch a possible graph with the following characteristics.

9. A 4th degree polynomial with a negative leading coefficient. The polynomial has one negative real zero greater than -3 and with a multiplicity of 2. The polynomial also has two distinct positive real zeros less than 7.



10. A 5th degree polynomial with a positive leading coefficient. The polynomial has two imaginary zeros. The polynomial has one positive real zero less than 4 with a multiplicity of 3.

don't show up on graph



IV. Use your calculator to answer the following questions about the given polynomial function.

11. $f(x) = x^4 + 13x^3 - 32x^2 - 26x + 60$

Possible Rational Roots:

$\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \pm 6, \pm 10, \pm 12, \pm 15, \pm 20, \pm 30, \pm 60$

ACTUAL Rational Roots: 2, -15

# of Real Roots: 4

# of Irrational Roots: 2

Irrational Roots: 2 ± 1.41
(Use the zero feature.)

# of Imaginary Roots: 0

Imaginary Roots: —

12. $f(x) = 2x^4 - 5x^3 + 11x^2 - 45x - 63$

Possible Rational Roots:

$\pm 1, \pm 3, \pm 7, \pm 9, \pm 21, \pm 63, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{7}{2}, \pm \frac{9}{2}, \pm \frac{21}{2}, \pm \frac{63}{2}$

ACTUAL Rational Roots: $\frac{7}{2}$, -1

# of Real Roots: 2

# of Irrational Roots: 0

Irrational Roots: —
(Use the zero feature.)

# of Imaginary Roots: 2

Imaginary Roots: $\pm 3i$

	2	-5	11	-45	-63
7	2	9	74	483	0
-1	2	-7	18	-63	0

	2	9	18	0
$\frac{7}{2}$	2	16	81	0
-1	2	-7	9	0

$0 = 2x^2 + 9x + 18$

$0 = 2x^2 + 9x + 9$

$x^2 + 4.5x = -9$

$x = -2.25 \pm 1.3i$