FUN	AP CALCULUS	
1	Topic: 2.8 Topic: 2.9	The Product Rule The Quotient Rule
Learning Objective FUN-3.B: Calculate derivatives of products and quotients of differentiable functions.		

Although the derivative of the sum (and difference) of two functions is simply the sum (or difference) of their derivatives, the **product and quotient** of two functions are handled **quite differently** when differentiating.

The Product Rule

A proof of the Product Rule can be seen on by following the Code.

> **The Product Rule**
> The product of two *differentiable* functions *f* and *g* is itself *differentiable.*
> Moreover, the derivative of $f \cdot g$ is the first function times the derivative of the second, plus the second function times the derivative of the first function.
> $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$

Example 1: Find the derivative of $h(x) = (3x - 2x^2)(5 + 4x)$.

Did we ***have*** to use the product rule for Example 1? What other way could we have found $h'(x)$?

Example 2: Find the derivative of each of the following.

a. $f(x) = x \sin x$

b. $g(x) = e^x x$

c. $h(x) = 2x \cos x - 2 \sin x$

d. $k(x) = x^2 \ln x$

The Quotient Rule

The Quotient Rule

The quotient, $\frac{f}{g}$ of two differentiable functions f and g is itself differentiable at all values for x for which $g(x) \neq 0$. Moreover, the derivative of $\frac{f}{g}$ is given by the denominator (bottom) times the derivative of the numerator(top) minus the numerator (top) times the derivative of the denominator(bottom), all divided by the denominator (bottom) squared.

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

A proof of the Quotient Rule can be seen on by following the Code

Example 3: Find the derivative of $y = \frac{5x+2}{x^2-1}$.

Example 4: Find the equation of the tangent line to $y = \frac{e^x}{1+x^2}$ at $\left(1, \frac{e}{2}\right)$.

Not every function that contains a quotient requires the use of the Quotient Rule.

Example 5: Complete the table.

Original Function	*Rewrite*	*Differentiate*	*Simplify*
a. $y = \frac{x^2+3x}{6}$			
b. $y = \frac{-3(3x-2x^2)}{7x}$			

Differentiation Using Data Tables

Example 6: The differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table below.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-4	10	3	3
2	3	-5	16	1
3	1	8	-4	-4
4	10	-1	1	2

a. Let $h(x) = f(x)g(x)$. Find $h'(3)$.

b. Find $\frac{d}{dx}\left[\sqrt{x} \cdot f(x)\right]$ when $x = 4$.

Example 7: The values of two differentiable functions, $f(x)$ and $g(x)$, along with their derivatives are given in the table below for several values of x.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	-1	1	2
2	4	-1	3	$\frac{3}{2}$
3	3	-1	4	1
4	2	-1	5	1
5	1	0	6	$-\frac{1}{2}$
6	2	1	4	-2

a.) Given $h(x) = \frac{f(x)}{g(x)}$, find $h'(2)$.

b.) Given $j(x) = \frac{g(x)}{\sqrt{x}}$, find $j'(4)$.