FUN	AP CALCULUS
1	Topic: 3.1 The Chain Rule
Learning Objective FUN-3.C: Calculate derivatives of compositions of differentiable functions.	

Let's introduce this new differentiation technique by showing you the types of functions that would require its use. The table below illustrates pairs of similar functions that can be differentiated with and without the Chain Rule.

The Chain Rule

$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$ OR $f'(u) \cdot u'$ OR $\frac{dy}{du} \cdot \frac{du}{dx}$

= Derivative of the outside function · Derivative of the inside function

= Derivative rule · Correction factor

Can be Differentiated WITHOUT the Chain Rule	Must be Differentiated WITH the Chain Rule
$y = x^2 + 1$	$y = \sqrt{x^2 + 1}$
$y = sin\, x$	$y = sin(6\, x)$
$y = 3x + 2$	$y = (3x + 2)^5$
$y = x + tan\, x$	$y = x + tan(x^2)$

Decomposing Composite Functions

Example 1: Complete the columns in the chart below.

	$y = f(g(x))$	$u = g(x)$ "Inside Function"	$y = f(u)$ "Outside Function"
a.	$y = \frac{1}{x+1}$		
b.	$y = sin(2\, x)$		
c.	$y = \sqrt{3x^2 - x + 1}$		
d.	$y = tan^2\, x$		

Example 2: Find $\frac{dy}{dx}$ for $y = (x^2 + 1)^3$

Example 3: Find $\frac{dy}{dx}$ for each of the following.

a. $y = \sqrt{(x^2 - 1)}$

b. $y = \frac{-7}{(2x-3)^2}$

Example 4: Find the derivative of each.

a. $y = cos(\,3x - 1)$

b. $y = sec(4\,x)$

Example 5: Find the derivative of each.
a. $y = ln(\,x^2 + 1)$

b. $y = (ln\,x)^3$

c. $\boldsymbol{y = 2^x}$

d. $\boldsymbol{y = 2^{3x}}$

e. $y = \log(\cos x)$

f. $y = e^{\frac{1}{x}}$

Chain Rule Differentiation Using Data Tables

Example 6: The values of two differentiable functions, $f(x)$ and $g(x)$, along with their derivatives are given in the table below for several values of x.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	-1	1	2
2	4	-1	3	$\frac{3}{2}$
3	3	-1	4	1
4	2	-1	5	1
5	1	0	6	$-\frac{1}{2}$
6	2	1	4	-2

a. Given $h(x) = f(g(x))$, find $h'(2)$.

b. Given $j(x) = g(f(x))$, find $j'(1)$.

Repeated Use of the Chain Rule

It is not uncommon for certain compositions of functions to require two uses of the Chain Rule.

$$y = \frac{-7}{(2x-3)^2}$$

Example 7: Find the derivative of each trigonometric function.

a. $y = sin^4(3\,x)$

b. $y = \sqrt{sec\left(\frac{x}{2}\right)}$

c. $y = cot^{\frac{2}{3}}\left(\frac{3x}{2}\right)$

The following properties from Algebra II will be useful in calculus when dealing with natural logarithms.

<u>**Logarithmic Properties**</u>

If *a* and *b* are positive numbers and *n* is rational, then the following properties are true.

1. $ln(1) = 0$
2. $ln(ab) = ln\, a + ln\, b$
3. $ln(a^n) = n\, ln\, a$
4. $ln\left(\frac{a}{b}\right) = ln\, a - ln\, b$

Example 8: $y = ln\left[\frac{x(x^2+1)^2}{\sqrt{2x^3-1}}\right]$.