

FUN	AP CALCULUS AB		
1	Topic: 3.2	Implicit Differentiation	
Learning Objective FUN-3.D: Calculate derivatives of implicitly defined functions.			

Explicit Functions vs Implicit Relations

Up to this point, you have learned how to differentiate functions that are expressed in an explicit form.

We will now take a look at relations written in an implicit form. Note the differences in the two types of functions from the table below.

<u>Explicitly Written Functions</u>	<u>Implicitly Written Equations</u>
$y = x^2 + 1$	$x^2 - 2y^3 + 4y = 2$
$f(x) = \sqrt{4x^3 - 2x + 1}$	$xy - 2x^2 + y^3 = x + y$

Example 1: Differentiating with Respect to x .

Find each derivative.

a. $\frac{d}{dx}[x^3] =$

b. $\frac{d}{dx}[y^3] =$

c. $\frac{d}{dx}[x + y] =$

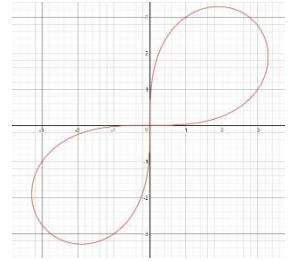
d. $\frac{d}{dx}[xy^2] =$

Guidelines for Implicit Differentiation

1. Differentiate both sides of the equation *with respect to* x .
2. Collect all terms involving $\frac{dy}{dx}$ on the left side of the equation and move all the other terms to the right side of the equation.
3. Factor $\frac{dy}{dx}$ out of the left side of the equation.
4. Solve for $\frac{dy}{dx}$ by dividing both sides of the equation by the left-hand factor that does not contain $\frac{dy}{dx}$.

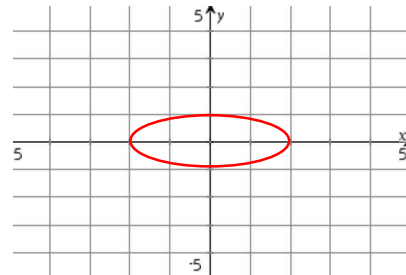
Example 2: Find $\frac{dy}{dx}$ for $y^3 + y^2 - 5y - x^2 = -4$

Example 3: Find the slope of the graph of $3(x^2 + y^2)^2 = 100xy$ at the point $(3, 1)$.



Example 4: Consider the equation $x^2 + 4y^2 = 4$.

a. Find $\frac{dy}{dx}$ for the equation above.



c. Find the slope of the tangent line to the curve at the point $(\sqrt{2}, \frac{-1}{\sqrt{2}})$.

d. Write the equation of the tangent line drawn to the curve at the point $(\sqrt{2}, \frac{-1}{\sqrt{2}})$.