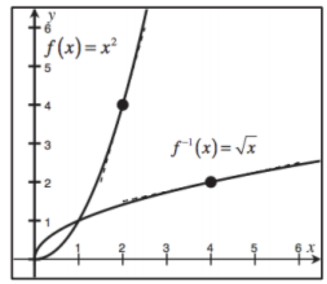
FUN	AP CALCULUS		
1	Topic: 3.3	Differentiating Inverse Functions	
Learning Objective FUN-3.E: Calculate derivatives of inverse and inverse trigonometric function			

Let's review Inverse function notation.

Now, suppose you were asked to show that the derivative of $f(x) = x^2, x > 0$ at the point $(2, 4)$ is the reciprocal of the derivative of the **inverse** to $f(x) = x^2, x > 0$ at the point $(4, 2)$. **See picture.**



MEMORIZE and UNDERSTAND

<u>The Derivative of an Inverse Function</u>
Let f be a function that is differentiable on an interval I. If f has an inverse function f^{-1}, then f^{-1} is differentiable at any x for which $f'(f^{-1}(x)) \neq 0$. Moreover, $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$

<u>Example 1</u>: If $f(x) = \sqrt{x+5}$, find the derivative of $f^{-1}(x)$ at $x = 3$.

Method 1:

<u>This works when it is easy to generate the inverse function</u>. Which on the AP Exam, is not often and usually never

a) Find the inverse function by interchanging *x* and *y* and solving for *y*.
b) Take the derivative of this new *y*. That will be the derivative of the inverse function.
c) Plug in your given *k* value (which is some value for *x*).

Let's verify our answer to **Example 1** using the formula $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$

Re – do Example 1: If $f(x) = \sqrt{x+5}$, find the derivative of $f^{-1}(x)$ at $x = 3$.

Example 2: Selected values of a strictly monotonic function $g(x)$ and its derivative $g'(x)$ are shown on the table below.

x	-3	-1	1	4
$g(x)$	5	1	0	-3
$g'(x)$	-4	$-\frac{1}{5}$	$-\frac{1}{6}$	-2

a. Find $(g^{-1})'(1)$

b. Find $(g^{-1})'(-3)$

Method 2:

This method is used when finding the inverse of the function is difficult or impossible.

You are given a function $f(x)$ and given an x-value on $f^{-1}(x)$.

a) Find the inverse function by interchanging *x* and *y*. (Don't solve for *y*)

b) Find $\frac{dy}{dx}$ implicitly

c) Solve for $\frac{dy}{dx}$. It will be in terms of *y*.

d) Replace the value of *x* on your inverse function from Step 'a' above and solve for *y*.

* If the y-value is not given or difficult to solve, you will need to use a calculator (if allowed) or trial and error to find it.

e) Plug that value of *y* into $\frac{dy}{dx}$.

Example 3: If $f(x) = x^3 + x$; Find $[f^{-1}(10)]'$

It is possible that Method 2 can be streamlined a bit by using the formula $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$.

<u>Re – do Example 3</u>: If $f(x) = x^3 + x$; Find $[f^{-1}(10)]'$ using the formula above.

What information can you tell me about $f(x)$ and $f^{-1}(x)$?

<u>Example 4</u>: Find the derivative of the inverse function of $f(x) = \frac{1}{4}x^3 + x - 1$ at $x = 3$ using the formula.

What information can you tell me about $f(x)$ and $f^{-1}(x)$?

<u>Example 5</u>: Find the derivative of the inverse function of $f(x) = x + sin\, x$ at $x = \pi$ using the formula.

What information can you tell me about $f(x)$ and $f^{-1}(x)$?