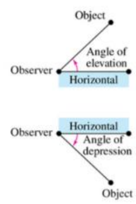
Chapter 4 Applications of Right Triangles

4.3 and 4.8 **Solving Right Triangles**

Many problems with right triangles involve the angle of elevation or the angle of depression. The angle of elevation from point X to point Y (above X) is the angle made by line XY and a horizontal line through X. **The angle of elevation is always measured from the horizontal!** The angle of depression from point X to point Y (below X) is the angle made by line XY and a horizontal line through X.

Figure 4.23



Example 1: Donna knows that when she stands 123 ft from the base of a flagpole, the angle of elevation to the top is $26°40'$. If her eyes are 5.3 ft above the ground, find the height of the flagpole.

Example 2: The length of the shadow of a building 34.09m tall is 37.62 m. Find the angle of elevation of the sun.

Example 3: Solve right triangle ABC, with $A = 34°30'$ and c=12.7.

Example 4: Solve right triangle ABC if a=29.43cm and c=53.588cm.

4.1

Arc Length

For a circle of radius r, a central angle θ intercepts an arc of length s given by

$$s = r\theta \qquad \text{Length of circular arc}$$

where θ is measured in radians. Note that if $r = 1$, then $s = \theta$, and the radian measure of θ equals the arc length.

Example 6: A circle has a radius of 18.2cm. Find the length of the arc cut by a central angle having the following measures.

a) $\frac{3\pi}{8}$

b) $144°$

Example 7: Reno, Nevada, is approximately due north of Los Angeles. The latitude of Reno is $40°$ N, while that of Los Angeles is 34°N. If the radius of the earth is 6400 km, find the north-south distance between the two cities.

Example 8: A rope is being wound around a drum with radius .8725 feet. How much rope will be wound around the drum if the drum is rotated through an angle of 39.72°?

Area of a Sector of a Circle

For a circle of radius r, the area A of a sector of the circle with central angle θ is

$$A = \frac{1}{2}r^2\theta$$

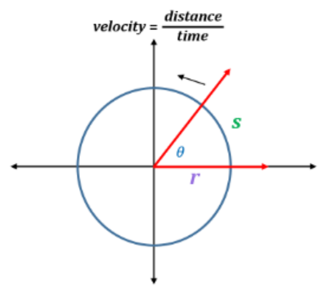
where θ is measured in radians.

Example 9: Suppose a field is in the shape of a sector of a circle. The central angle is $15°$ and the radius of the circle is 321m. Find the area of the field.

4.1 Linear and Angular Velocity

The measure of how fast *s* is changing is called ________________________ or how fast a point is moving on the circumference of the circle.

The measure of how fast θ is changing is called ________________________ or how fast the radian measure is changing.



Angular Velocity	Linear Velocity
$\omega = \frac{\theta}{t}$ ω in radians per unit time θ in radians	$v = \frac{s}{t}$ $v = \frac{r\theta}{t}$ $v = r\omega$

We always use ***radians*** as the unit of measure for θ. However, sometimes we will give **"angular velocity"** in layman's terms, such as *revolutions per minute* or *degrees per second*, to give a better understanding of how fast the angle is moving. So, to make calculations with the above formulas a unit conversion my need to occur in **Step 1**.

NOTE: Radians is an "invisible" unit. For example, $\theta = \frac{3\pi}{4}(rads)$ OR $r = 10\ ft$, by definition of a radian $r = \frac{10ft}{(1\ radian)}$.
Therefore, **linear velocity** = $\frac{(rads)}{second}\ X\ \frac{ft}{(rads)} = \frac{ft}{second}$ which is a *unit of length/unit of time*.

STEPS FOR SOLVING:

1. Write down the given. Include variables and their correct units (convert, if necessary).
   Also, indicate the unknown(?)
2. Pick an equation that relates all the variables to each other
3. If necessary, rearrange the equation for the unknown(?).
4. Put the given info including **UNITS** into a "bridge" using the equation in Step 3.
5. Show how the units cancel and carefully plug the values into your calculator, if necessary.
6. Given your final answer including **UNITS**

Example 1: A cylinder with a 2.5 ft radius is rotating 120 rpm
(a) Give the angular velocity in rad/sec and in degrees per second
(b) Find the linear velocity of a point on its rim in mph

Example 2: A tire with a 9-inch radius is rotating at 30 mph. Find the angular velocity of a point on it rim. Also express the result in revolutions per minute.

Example 3: The two pulleys connected by a belt have radii of 15 cm and 8 cm. The larger pulley rotates 24 times in 36 seconds.

(a) Find the angular velocity of the small pulley in radians per second

(b) Find the linear velocity of a point on the belt that connects the two pulleys in centimeters per second

4.8 Further Applications of Right Triangles

Other applications of right triangles involve _____________, an important idea in navigation. There are two common ways to express bearing. When a single angle is given, such as , it is understood that the bearing is measured in a clockwise direction from due north.

Sketch a direction for each of the following:

$32°$ $164°$ $304°$

Example 1: Radar stations A and B are on an east-west line, 3.7 km apart. Station A detects a plane at C, on a bearing of $61°$. Station B simultaneously detects the same plane on a bearing of $331°$. Find the distance from A to C.

The other common system for expressing bearing starts with a north-south line and uses an acute angle to show the direction, either east or west, from this line.

Sketch a direction for each of the following:

$N42°E$ $S40°W$ $N52°W$

Example 2: The bearing from A to C is $S52°E$. The bearing from A to B is $N84°E$. The bearing from B to C is $S38°W$. A plane flying at 250 mph takes 2.4 hours to go from A to B. Find the distance from A to C.

Example 3: Francisco needs to know the height of a tree. From a given point on the ground he finds that the angle of elevation to the top of the tree is $36°40'$. He then moves back 50 feet. From the second point, the angle of elevation to the top of the tree is $22°10'$. Find the height of the tree.