Topic 4.2 – Straight-Line Motion: Connecting Position, Velocity and Acceleration

A particle is moving along a horizontal line with position function as given. Perform an analysis of the particle's direction, acceleration, motion (speeding up or slowing down), and position by completing the given number lines.

1.) $s(t) = -t^3 + 9t^2 - 24t + 1$

$s'(t) = v(t) = -3t^2 + 18t - 24$

$v(t) = 0$ when $-3(t^2 - 6t + 8) = 0$

$-3(t-2)(t-4) = 0$

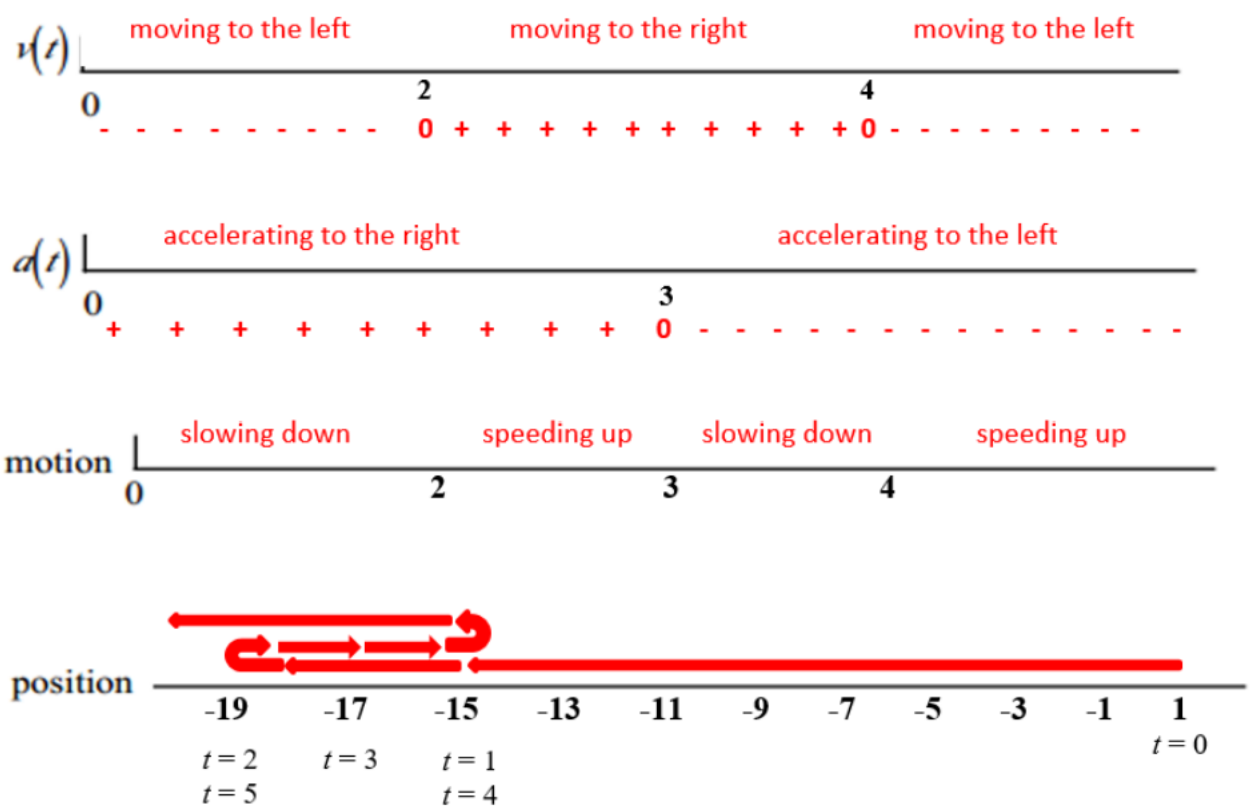
$t = 2,\ t = 4$

$s''(t) = v'(t) = a(t) = -6t + 18$

$a(t) = 0$ when $-6(t-3) = 0$

$t = 3$

$s(0) = 1$, $s(1) = -15$, $s(2) = -19$, $s(3) = -17$, $s(4) = -15$, $s(5) = -19$, $s(6) = -35$, $s(7) = -69$



2.) $s(t) = t + \dfrac{9}{t+1} + 1$

$s(0)=10 \quad s(1)=6.5 \quad s(2)=6 \quad s(3)=6.25 \quad s(4)=6.8 \quad s(5)=7.5 \quad s(6)\approx 8.286$

$$s'(t) = v(t) = 1 - 9(t+1)^{-2} = 1 - \frac{9}{(t+1)^2}$$

$$= \frac{(t+1)^2 - 9}{(t+1)^2} = \frac{t^2+2t-8}{(t+1)^2}$$

$v(t) = 0$ when $t^2 + 2t - 8 = 0$

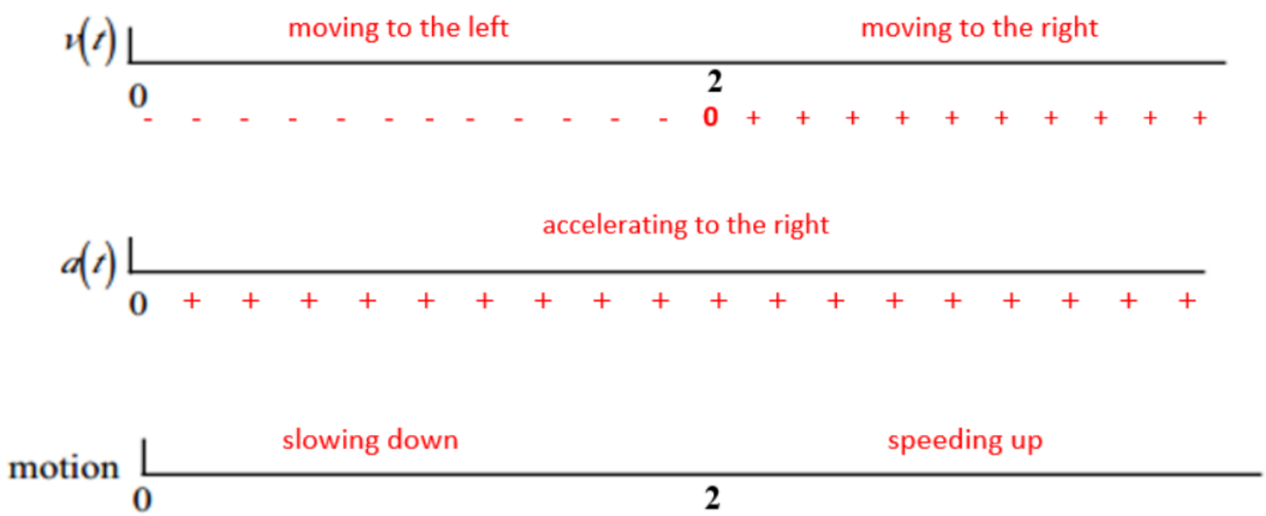
$$(t+4)(t-2) = 0$$

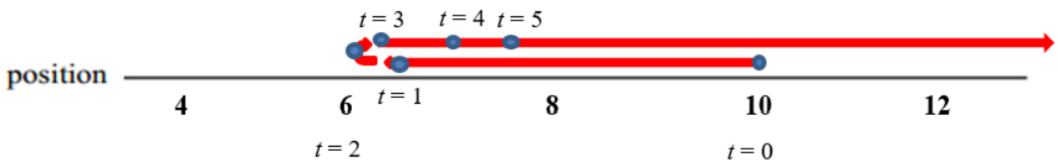
$$t = 2$$

$$s''(t) = v'(t) = a(t) = \frac{(2t+2)(t+1)^2 - (t^2+2t-8)\cdot 2(t+1)}{(t+1)^2}$$

$$= \frac{2t^3+6t^2+6t+2-(2t^3+6t^2-12t-16)}{(t+1)^2} = \frac{18t+18}{(t+1)^2}$$

$a(t) = 0$ when $18(t+1) = 0$ which will never happen in our domain of $t > 0$.





3.) A 45-caliber bullet fired straight up from the surface of the moon would reach a height of $s = 832t - 2.6t^2$ feet after t seconds. On Earth, in the absence of air, its height would be $s = 832t - 16t^2$ feet after t seconds. How long would it take the bullet to hit the ground in either case?

Earth	**Moon**
The bullet hits the ground when $s = 0$ $832t - 16t^2 = 0$ $-16t(t-52) = 0$ $t = 52$ The bullet will hit the ground after 52 seconds.	The bullet hits the ground when $s = 0$ $832t - 2.6t^2 = 0$ $-2.6t(t-320) = 0$ $t = 320$ The bullet will hit the ground after 320 seconds.

4.) A dynamite blast propels a heavy rock straight up with a launch velocity of 160 ft/sec (about 109 mph). The rock reaches a height $s(t) = 160t - 16t^2$ feet after t seconds.

a.) How high does the rock go?

$v(t) = 160 - 32t$

$160 - 32t = 0$

$32t = 160$

$t = 5$

$s(5) = 160(5) - 16(5)^2$

$= 400$

The rock reaches a height of 400 feet.

b.) What is the velocity of the rock when it is 256 ft above the ground

i.) on the way up?

$256 = 160t - 16t^2$

$16t^2 - 160t + 256 = 0$

$16(t^2 - 10t + 16) = 0$

$16(t-8)(t-2) = 0$

$t = 2,\ t = 8$

$v(2) = 160 - 32(2)$

$= 96 \text{ ft/sec}$

ii.) on the way down?

$v(8) = 160 - 32(8)$

$= -96 \text{ ft/sec}$

c.) What is the acceleration of the rock at any time t during its flight (after the blast)?

$a(t) = -32 \text{ ft/sec}^2$

d.) When does the rock hit the ground?

$160t - 16t^2 = 0$

$16t(10 - t) = 0$

$t = 0,\ t = 10$

The rock hits the ground after 10 seconds.

5.) Uncle Si's four-wheeler runs out of gas as it goes up a hill. The vehicle rolls to a stop then starts rolling backwards. As it rolls, its displacement $d(t)$ in feet from the bottom of the hill at t seconds since the vehicle ran out of gas is given by $d(t) = 145 + 31t - t^2$.

a.) How far from the bottom of the hill was Uncle Si when he ran out of gas?

When the four-wheeler runs out of gas, $t = 0$.

$d(0) = 145 + 31(0) - (0)2$

$= 145$ feet

b.) When is his velocity positive? What does this mean in the context of the problem?

$v(t) = 31 - 2t$

$0 = 31 - 2t$

$t = 15.5$

The velocity is positive on the time interval $(0, 15.5)$. This means Si was moving up the hill from time $t = 0$ to $t = 15.5$.

(Think of this as the momentum.)

c.) How far was the four-wheeler from the bottom of the hill when it starts to roll backwards?

$d(15.5) = 145 + 31(15.5) - (15.5)^2$

$= 385.25$ feet

d.) If Si keeps his foot off the brake, how long will it take for him to be at the bottom of the hill? What will his speed be at that time?

$145 + 31t - t^2 = 0$

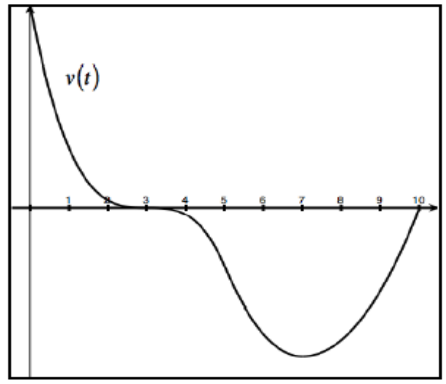
Using the calculator, we get

$t = -4.12779$ and $t = 35.1278$

$|v(35.1278)| = |31 - 2(35.1278)| = 39.256$

It will take Si 35.128 seconds to coast to the bottom of the hill at which point his speed will be 39.256 feet per second.

6.) The velocity $v(t)$ of a particle moving along the x-axis is shown in the figure to the right with t measured in seconds. Later in this course, you will learn ways to justify each response as well as finding how far the particle traveled.



a.) At what time t is the particle farthest to the right?

The particle is farthest to the right at time $t = 3$. The reason for this is due to the fact that the graph of $v(t)$ is positive (lies above the $x-$axis to the left of $t = 3$ prior to becoming and remaining negative until time $t = 10$.

b.) At what time intervals is the particle speeding up?

The particle is speeding up on the time interval (3,7). This is because the sign of $v(t)$ is negative while the slope of the tangent to $v(t)$, which represents $a(t)$ is negative as well. Therefore $v(t)$ and $a(t)$ have the same sign. (On no other interval are the signs of $v(t)$ and $a(t)$ the same.