Topic 4.3 – Rates of Change in Applied Contexts Other Than Motion

Answer each of the following problems. A **calculator** may be used and is required for most of the problems.

1. A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \frac{600t}{t+3}, & 0 \le t \le 5 \\ 1000e^{-0.2t}, & t > 5 \end{cases}$$

Find $r'(3)$. Using correct units, explain the meaning of the value in the context of this problem.

2. The Apple® Store is having a 12-hour sale. The total number of shoppers who have entered the store t hours after the sale begins is modeled by the function S defined by $S(t) = 0.5t^4 - 16t^3 + 144t^2$ for $0 \le t \le 12$. At time $t = 0$, when the sale begins, there are no shoppers in the store.

a. At what rate are shoppers entering the store 3 hours after the start of the sale?

b. Let the function L model the total number of shoppers who have exited the store t hours after the sale began. The function L is defined by $L(t) = -80 + \frac{4400}{t^2 - 14t + 55}$ for $0 \le t \le 12$. Is the number of shoppers in the store increasing or decreasing at time $t = 3$? Justify your answer.

3. The tide removes sand from Sandy Point Beach at a rate modeled by the function *R*, given by $R(t) = -2 + 5\,sin\left(\frac{4\pi t}{25}\right)$. A pumping station adds sand to the beach at a rate modeled by the function *S*, given by $S(t) = \frac{15t}{1+3t}$. Both *R*(*t*) and *S*(*t*) are measured in cubic yards per hour and *t* is measured in hours for $0 \le t \le 6$. At time *t* = 0, the beach contains 2500 cubic yards of sand.

a. Find the rate at which the total amount of sand on the beach is changing at time *t* = 4.

4. The rate at which rainwater flows into a drainpipe is modeled by the function *R*, where $R(t) = 20\,sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, *t* is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at *t* = 0.

a. Is the amount of water in the pipe increasing or decreasing at time *t* = 3 hours? Give a reason for your answer.