

### Topic 4.3 – Rates of Change in Applied Contexts Other Than Motion

Answer each of the following problems. A **calculator** may be used and is required for most of the problems.

1. A 12,000-liter tank of water is filled to capacity. At time  $t = 0$ , water begins to drain out of the tank at a rate modeled by  $r(t)$ , measured in liters per hour, where  $r$  is given by the piecewise-defined function

$$r(t) = \begin{cases} \frac{600t}{t+3}, & 0 \leq t \leq 5 \\ 1000e^{-0.2t}, & t > 5 \end{cases}$$

Find  $r'(3)$ . Using correct units, explain the meaning of the value in the context of this problem.

$r'(3) = 50$   
 The rate at which the water is draining from the tank is increasing by 50 liters/hr per hour at time  $t = 3$  hours.

2. The Apple® Store is having a 12-hour sale. The total number of shoppers who have entered the store  $t$  hours after the sale begins is modeled by the function  $S$  defined by  $S(t) = 0.5t^4 - 16t^3 + 144t^2$  for  $0 \leq t \leq 12$ . At time  $t = 0$ , when the sale begins, there are no shoppers in the store.



- a. At what rate are shoppers entering the store 3 hours after the start of the sale?

$S'(3) = 486$   
 The number of shoppers entering the Apple store is increasing at a rate of 486 shoppers/hour at time  $t = 3$  hours.

- b. Let the function  $L$  model the total number of shoppers who have exited the store  $t$  hours after the sale began. The function  $L$  is defined by  $L(t) = -80 + \frac{4400}{t^2 - 14t + 55}$  for  $0 \leq t \leq 12$ . Is the number of shoppers in the store increasing or decreasing at time  $t = 3$ ? Justify your answer.

$A'(t) = S'(t) - L'(t)$   
 $A'(3) = S'(3) - L'(3)$   
 $A'(3) \hat{=} 486 - 72.7273 \approx 413.2727$   
 The number of shoppers in the Apple store is increasing at time  $t = 3$  hours because  
 $A'(3) > 0$  or  $S'(3) > L'(3)$

3. The tide removes sand from Sandy Point Beach at a rate modeled by the function  $R$ , given by  $R(t) = -2 + 5 \sin\left(\frac{4\pi t}{25}\right)$ . A pumping station adds sand to the beach at a rate modeled by the function  $S$ , given by  $S(t) = \frac{15t}{1+3t}$ . Both  $R(t)$  and  $S(t)$  are measured in cubic yards per hour and  $t$  is measured in hours for  $0 \leq t \leq 6$ . At time  $t = 0$ , the beach contains 2500 cubic yards of sand.



a. Find the rate at which the total amount of sand on the beach is changing at time  $t = 4$ .

$$A'(t) = S(t) - R(t)$$

$$A'(4) = S(4) - R(4) \approx 4.61538 - \cancel{2.09128} \approx 2.5241$$

$\approx 2.0912$   
 $\text{yd}^3/\text{hr}$

4. The rate at which rainwater flows into a drainpipe is modeled by the function  $R$ , where  $R(t) = 20 \sin\left(\frac{t^2}{35}\right)$  cubic feet per hour,  $t$  is measured in hours, and  $0 \leq t \leq 8$ . The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by  $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$  cubic feet per hour, for  $0 \leq t \leq 8$ . There are 30 cubic feet of water in the pipe at  $t = 0$ .



a. Is the amount of water in the pipe increasing or decreasing at time  $t = 3$  hours? Give a reason for your answer.

$$A'(t) = R(t) - D(t)$$

$$A'(3) = R(3) - D(3) \approx 5.0864 - 5.4 \approx -0.3136$$

The amount of water in the pipe is decreasing at time  $t = 3$  hours because  $A'(3) < 0$  or  $R(3) < D(3)$