Skill Builder: Topics 4.4 & 4.5 – Related Rates

1. A rectangular well is 6 feet long, 4 feet wide, and 8 feet deep. If water is running into the well at the rate of $3\,\text{ft}^3/\text{sec}$, find how fast the water is rising (keep in mind which variables are constant and which are changing).

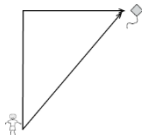
2. A spherical hot air balloon is being inflated. Air is blown into the balloon at the rate of $2\,\text{ft}^3/\text{sec}$.
   a. How fast is the radius of the balloon changing when the radius is 3 feet?
   b. How fast is the surface area of the balloon changing at the same time?

3. A 12-foot ladder stands against a vertical wall. The lower end of the ladder is being pulled away from the wall at the rate of 2 ft/sec
   a. How fast is the top of the ladder moving down the wall at the instant it is 6 feet above the ground?
   b. How fast is the angle of elevation of the ladder changing at the same instant?

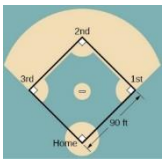
4. Superman is in level flight 6 miles above the ground. His flight plan takes him directly over and past Eisenhower High School. How fast is he flying when the distance between him and EHS is exactly 10 miles and this distance is increasing at the rate of 40 mph?

5. A boy flies a kite which is 120 ft directly above his hand. If the wind carries the kite horizontally at the rate of 30 ft/min, at what rate is the string being pulled out when the length of the string is 150 ft?

6. The same boy flies a kite which is now 100 ft above the ground. If the string is being pulled out at the rate of 10 ft/sec because the wind carries the kite horizontally directly away from the boy, what is the rate of change of the angle the kite makes with the vertical when the angle is 30°?



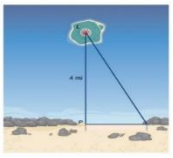
7. A baseball diamond is a 90-foot square. A ball is batted along the third-base line at a constant rate of 100 feet per second.



   a. How fast is its distance changing from first base at the time when the ball is halfway to 3rd base?

   b. How fast is its distance changing from first base at the time when the ball reaches 3rd base?

8. A lighthouse, *L*, is on an island 4 miles away from the closest point, *P*, on the beach as shown in the image below. If the beam of light (rotating clockwise) from the lighthouse is moving at a rate of 2 miles/min along the shoreline, what is the angular velocity of the light at the time when the beam of light is 1 mile away from the point *P*.

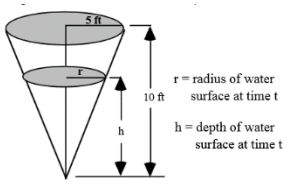


9. A lighthouse, *L*, is on an island 4 miles away from the closest point, *P*, on the beach as shown in the image above. If the lighthouse light rotates clockwise at a constant rate of 10 revolutions/min, how fast does the beam of light move across the beach 2 miles away from the point *P*.

10. How fast does the radius of a spherical soap bubble change when you blow air into it at the rate $10\ \text{cm}^3$ / sec at the time when the radius is 2 cm?

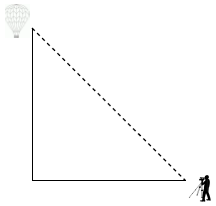
11. How fast does the water level drop when a cylindrical tank of radius 6 feet is drained at the rate of $3\ \text{ft}^3/min$?

12. Water runs into a conical tank at the rate of $9\ \text{ft}^3/min$. The tank stands vertex down and has a height of 10 feet and a base radius of 5 feet. How fast is the water level rising when the water is 6 feet deep?

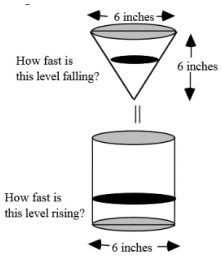


13. Kyler is filling an ice cream cone from a soft-serve ice cream machine. The cone is 12 cm tall and has a radius of 4 cm. If the ice cream machine fills the cone evenly at a constant rate of $1.5\ \text{cm}^3$/sec, what is the rate at which the height is changing when the height is 5 cm?

14. A hot air balloon, rising straight up from a level field, is tracked by a range finder 500 feet from the lift-off point.
    a. At the moment the range finder's elevation angles is $\frac{\pi}{4}$, the angle is increasing at the rate of 0.14 radians/min. How fast is the balloon rising?
    b. If the balloon rises at 100 ft/min, when the angle of elevation is $45°$, how fast is the angle changing?



15. Coffee is draining from a conical filter into a cylindrical coffeepot at the rate of $10\ \text{in}^3/min$.
    a. How fast is the level of coffee in the pot rising when the coffee in the filter is 5 inches deep?
    b. How fast is the level of coffee in the cone falling at that moment?



16. A particle is moving along a curve whose equation is $\frac{xy^3+x}{1+y^2} = \frac{9}{5}$. Assume the *x*-coordinate is increasing at the rate of 6 units/sec when the particle is at the point (1,2). At what rate is the *y*-coordinate of the point changing at that instant. Is it rising or falling?