

## Related Rates Practice Problems AP Calculus BC

- A conical tank has a height that is always 3 times its radius. If water is leaving the tank at the rate of 50 cubic feet per minute, how fast is the water level falling in feet per minute when the water is 3 feet high?  $\frac{50}{\pi} \text{ ft/min}$
- Water pours into a conical tank at a constant rate of 10 cubic feet/min. The tank is 10 ft tall and at its widest, has a radius of 4 feet. How fast is the water level rising when it is 5 feet high?  $5/2\pi \text{ ft/min}$
- A liquid is being drained from a conical tank at the rate of 8 cubic feet per minute. If both the height and the radius at the top of the tank are 12 feet, how fast is the level of liquid dropping when it is 3 feet deep?  $8/9\pi \text{ ft/min}$
- A rectangular well is 6 feet long, 4 feet wide, and 8 feet deep. If water is running into the well at the rate of  $3 \text{ ft}^3/\text{sec}$ , find how fast the water is rising (keep in mind which variables are constant and which are changing).  $1/8 \text{ ft/sec}$
- A spherical hot air balloon is being inflated. If air is blown into the balloon at the rate of  $2 \text{ ft}^3/\text{sec}$ ,
  - find how fast the radius of the balloon is changing when the radius is 3 feet.  $1/8\pi \text{ ft/sec}$
  - find how fast the surface area is increasing at the same time.  $S.A = 4\pi r^2$   
 $4/3 \text{ ft}^2/\text{sec}$
- A 12 foot ladder stands against a vertical wall. If the lower end of the ladder is being pulled away from the wall at the rate of  $2 \text{ ft/sec}$ ,
  - how fast is the top of the ladder coming down the wall at the instant it is 6 feet above the ground?  $\sqrt{108}/3 \text{ ft/sec}$
  - how fast is the angle of the elevation of the ladder changing at the same instant? Decreasing  $1/3 \text{ rad/sec}$
- Superman is in level flight 6 miles above ground. His flight plan takes him directly over Wissahickon High. How fast is he flying when the distance between him and WHS is exactly 10 miles and this distance is increasing at the rate of 40 mph?  $50 \text{ mph}$
- Suppose that liquid is to be cleared of sediment by allowing it to drain through a conical filter that is 16 cm high and has a radius of 4 cm at the top. Liquid is forced out of the cone at a constant rate of  $\frac{2\text{cm}^3}{\text{min}}$ . At what rate is the depth of the liquid changing at the instant when the liquid in the cone is 8 cm deep? Decreasing  $1/2\pi \text{ cm/min}$
- A boy flies a kite which is 120 ft directly above his hand. If the wind carries the kite horizontally at the rate of  $30 \text{ ft/min}$ , at what rate is the string being pulled out when the length of the string is 150 ft?  $18 \text{ ft/min}$
- The same boy flies a kite which is now 100 feet above the ground. If the string is pulled out at the rate of 10 ft/sec because the wind carries the kite horizontally directly away from the boy, what is the rate of change of the angle the kite makes with the vertical when the angle is  $30^\circ$ .
- A baseball diamond is a 90-foot square. A ball is batted along the third-base line at a constant rate of 100 feet per second. How fast is its distance changing from first base at the time when a) the ball is halfway to 3rd base and b) it reaches 3rd base.
  - $44.721 \text{ ft/sec}$
  - $70.711 \text{ ft/sec}$
- A plane is flying west at  $500 \text{ ft/sec}$  at an altitude of 4,000 ft. The plane is tracked by a searchlight on the ground. If the light is to be trained on the plane, find the change in the angle of elevation of the searchlight at a horizontal distance of 2,000 ft.  $\frac{d\theta}{dt} = \frac{1}{10} \text{ rad/sec}$

13. A revolving light located 5 miles from a straight shore line turns with a constant angular velocity. What velocity does the light revolve if the light moves along the shore at the rate of 15 miles per minute when the beam makes an angle of  $60^\circ$  with the shore line?  $3/4 \text{ rads/min}$
14. How fast does the radius of a spherical soap bubble change when you blow air into it at the rate of  $10 \text{ cm}^3/\text{sec}$  at the time when the radius is 2 cm?  $5/8\pi \text{ cm/sec}$
15. How fast does the water level drop when a cylindrical tank of radius 6 feet is drained at the rate of  $3 \text{ ft}^3/\text{min}$ ?  $1/12\pi$
16. A hot air balloon, rising straight up from a level field, is tracked by a range finder 500 feet from the lift-off point. At the moment the range finder's elevation angle is  $\frac{\pi}{4}$ , the angle is increasing at the rate of 0.14 radians/min. How fast is the balloon rising?  $140 \text{ ft/min}$
17. Water runs into a conical tank at the rate of  $9 \text{ ft}^3/\text{min}$ . The tank stands vertex down and has a height of 10 feet and a base radius of 5 feet. How fast is the water level rising when the water is 6 feet deep?  $1/\pi \text{ ft/min}$
18. Two truck convoys leave a depot at the same time. Convoy A travels east at 40 mph and convoy B travels north at 30 mph. How fast is the distance between the convoys changing a) in 6 minutes b) in 30 minutes  
a) 50 mph b) 50 mph
19. Two commercial jets at 40,000 ft. are both flying at 520 mph towards an airport. Plane A is flying south and is 50 miles from the airport while Plane B is flying west and is 120 miles from the airport. How fast is the distance between the two planes changing at this time?  $680 \text{ mph}$
20. A spherical Tootsie Roll Pop that you are enjoying is giving up volume at a steady rate of  $0.25 \text{ in}^3/\text{min}$ . How fast will the radius be decreasing when the Tootsie Roll Pop is .75 inches across? Decreasing  $0.141 \text{ in/min}$
21. The mechanics at Toyota Automotive are reboring a 6-inch deep cylinder to fit a new piston. The machine that they are using increases the cylinder's radius one-thousandth of an inch every 3 minutes. How rapidly is the cylinder volume increasing when the bore (diameter) is 3.80 inches?  $0.024 \text{ in}^3/\text{min}$
22. Sand falls at the rate of  $30 \text{ ft}^3/\text{min}$  onto the top of a conical pile. The height of the pile is always  $\frac{3}{8}$  of the base diameter. How fast is the height changing when the pile is 12 ft. high? Rising  $15/28\pi \text{ ft/min}$
23. A rowboat is pulled toward a dock from the bow through a ring on the dock 6 feet above the bow. If the rope is hauled in at 2 ft/sec, how fast is the boat approaching the dock when 10 feet of rope are out?  $5/2 \text{ ft/sec}$
24. A particle is moving along the curve whose equation is  $\frac{y^3}{1+y^2} = \frac{8}{5}$ . Assume the x-coordinate is increasing at the rate of 6 units/sec when the particle is at the point (1, 2). At what rate is the y-coordinate of the point changing at that instant. Is it rising or falling? Falling  $60/7 \text{ units/sec}$
25. A balloon is rising vertically above a level, straight road at a constant rate of 1 foot/sec. Just when the balloon is 65 feet above the ground, a bicycle passes under it going 17 feet/sec. How fast is the distance between the bicycle and balloon increasing 3 seconds later?  $11 \text{ ft/sec}$