$y = 2^x$

$\log_2 y = x$

$\log_2 x = y$

Pre-Calculus Notes Name: Key

Section 3.2 – Logarithmic Functions

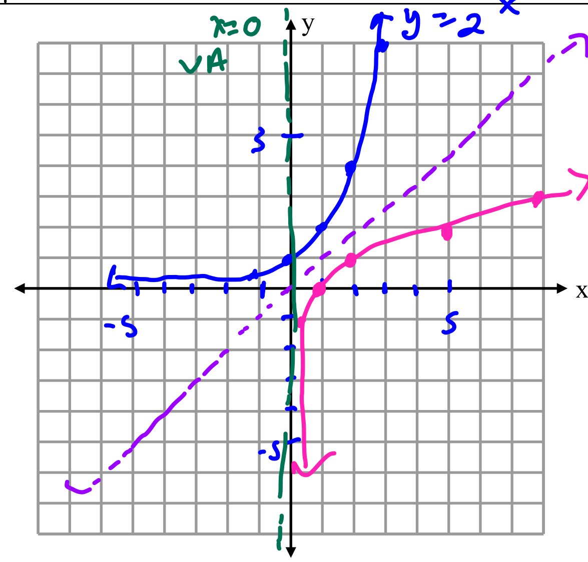
Since the exponential function $f(x) = b^x$ is one-to-one, it has an inverse function. The inverse function of an exponential function is called a logarithmic function.

MEMORIZE: If $x = b^y$ (exponential), then $y = \log_b x$ (log). $\log_b x = y$ **AND** If $y = \log_b x$ (log), then $x = b^y$ (exponential). $b^y = x$

EXAMPLE 1: Graph the function in ONE COLOR. Then graph its INVERSE in a SECOND COLOR.

ORIGINAL FUNCTION: $y = 2^x$
Domain: $(-\infty, +\infty)$ Range: $(0, +\infty)$
X-Intercepts: None
Y-Intercepts: 1
Increasing or Decreasing? Increasing
Equation of Asymptote: $y = 0$

INVERSE FUNCTION: $\log_2 x = y$
Domain: $(0, +\infty)$ Range: $(-\infty, +\infty)$
X-Intercepts: 1
Y-Intercepts: None
Increasing or Decreasing? Increasing
Equation of Asymptote: $x = 0$ (V.A)



MEMORIZE: $\log_{10} x = \log x$

A logarithm with a base of 10 is a common logarithm. So, instead of writing $\log_{10} x$, we will write $\log x$.

A logarithm with a base of "e" is a natural logarithm. So, instead of writing $\log_e x$, we will write $\ln x$.

$\log_e x = \ln x$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e \text{ and } e \approx 2.718281828...$$

A logarithm is an exponent

To what exponent do I raise base "b" to get a?

$\log_b a = x$ ← exponent

read as "log base b of a equals x"

Example 2: Rewrite each expression in logarithmic form.

a. $4^3 = 64$	b. $10^3 = 1000$	c. $e^{-2} \approx 0.14$
$\log_4 64 = 3$	$\log_{10} 1000 = 3$	$\log_e 0.14 = -2$ $\ln 0.14 = -2$

A log is an exponent

Example 3: Rewrite each expression in exponential form.

a. $\ln 2 \approx 0.70$ $e^{0.70} \approx 2$	b. $\log_5 125 = 3$ $5^3 = 125$	c. $\log 0.1 = -1$ $10^{-1} = 0.1$

Example 4: Use the definition of logarithmic function to evaluate each logarithm. NO CALCULATOR!

a. $\log_2 32 = x$ $2^x = 32$ $2^x = 2^5$ $x = 5$	b. $\log_3 1 = x$ $3^x = 1$ $x = 0$	c. $\log_4 2 = x$ $4^x = 2$ $(2^2)^x = 2^1$ $2^{2x} = 2^1$ $2x = 1$ $x = 1/2$	d. $\log_{10} \frac{1}{100} = x$ $10^x = \frac{1}{100}$ $10^x = 10^{-2}$ $\boxed{x = -2}$

To Solve a log equation, change it to exponential form

Example 5: Evaluate with the calculator. Round to 3 decimal places.

a. $\log 25$ ≈ 1.398	b. $\ln 0.34$ ≈ -1.079	c. $\log x = 2.014$ $10^{2.014} = x$ $x \approx 103.276$	d. $\ln x = -4$ $e^{-4} = x$ $x \approx 0.018$	e. $\log x = 0$ $10^0 = x$ $x = 1$

MEMORIZE: Change of Base Formula

The Change of Base Formula is used in order to evaluate a logarithm with a base other than 10 in the calculator. The Change-of-Base Formula is $\log_b x = \dfrac{\log_a x}{\log_a b}$

Example 6: Use the change of base formula to evaluate to 3 decimal places.

a. $\log_2 15$ $\dfrac{\log 15}{\log 2} \approx 3.907$	b. $\log_{\frac{1}{4}} 20$ $\dfrac{\log 20}{\log \frac{1}{4}} \approx -2.161$	c. $\log_{\sqrt{6}} 1.5$ $\dfrac{\ln 1.5}{\ln \sqrt{6}} \approx 0.453$

$\dfrac{\ln 15}{\ln 2} \approx 3.907$

$\dfrac{\log 1.5}{\log \sqrt{6}}$