

Properties of Logarithms

$$\log_b M \cdot N = \log_b M + \log_b N$$

$$\log_b \frac{M}{N} = \log_b M - \log_b N$$

$$\log_b M^x = x \cdot \log_b M$$

Simplifying Logarithms

Write each logarithmic expression as a single logarithm.

$$\begin{aligned} \log_4 64 - \log_4 16 &= \log_4 \frac{64}{16} \\ &= \log_4 4 \end{aligned}$$

Examples:

$$6 \log_5 x + \log_5 y =$$

$$3 \log 7 + \log x - 6 \log y =$$

$$\frac{1}{2}(\log_x 4 + \log_x y) - 3 \log_x 2 =$$

Expanding Logarithms

Expand each logarithm.

$$\log_2 7b = \log_2 7 + \log_2 b$$

$$\begin{aligned}\log_7 a^3 b^4 &= \log_7 a^3 + \log_7 b^4 \\ &= 3 \cdot \log_7 a + 4 \cdot \log_7 b\end{aligned}$$

Examples:

$$\log\left(\frac{y}{3}\right)^2 =$$

$$\log 2(x + 5)^2 =$$

$$\log_3 \sqrt[4]{\frac{x^2}{y}} =$$

$$\log_b \frac{\sqrt{x} \sqrt[3]{y^2}}{\sqrt[5]{z^2}}$$

Examples

Use the properties of logarithms to evaluate each expression.

$$3 \log_2 2 - \log_2 4$$

$$\log 1 + \log 100$$

Application: Sound Intensity (decibels)

$$L = 10 \log \frac{I}{10^{-12}} \quad \begin{array}{l} L = \text{apparent loudness} \\ I = \text{intensity of sound} \end{array}$$

Find the loudness of a chainsaw with an intensity of sound = 1.6×10^{-3}

$$L = 10 \log \frac{1.6 \times 10^{-4}}{10^{-12}}$$

$$L = 92 \text{ db}$$

Another chainsaw has an intensity of sound = 5.4×10^{-2}

How much louder/quieter is this chainsaw?