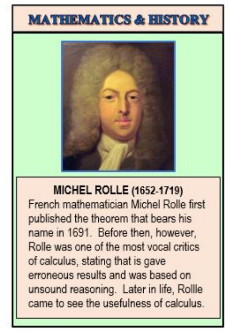
FUN		
3	Topic: 5.1	Using the Mean Value Theorem
Learning Objective FUN-1.B: Justify conclusions about functions by applying the Mean Value Theorem over an interval.		

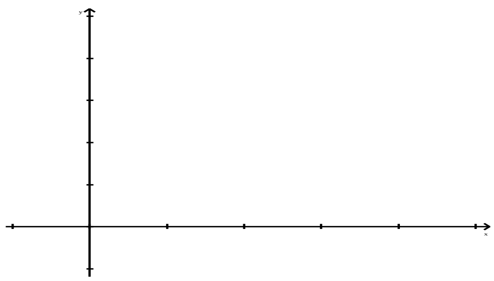
Before we introduce the main topic of the section, The Mean Value Theorem, let us start with a special case of this important theorem, Rolle's Theorem.



Rolle's Theorem

Activity

Step 1: Place two points anywhere on the coordinate plane below that have the same *y*-values.
Step 2: Connect the two points with a <u>continuous</u> function that is also <u>differentiable</u>.



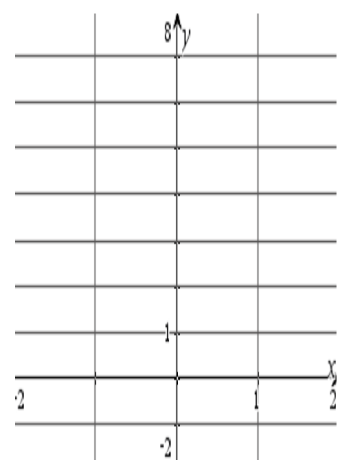
Conclusion: There MUST be at least one point on your function where you can draw a tangent line that is horizontal. (i.e. the slope is zero)

This activity basically models an important concept called **Rolle's Theorem.**

<u>Rolle's Theorem</u>

Let f be a function that is continuous on the closed interval $[a, b]$and differentiable on the open interval (a, b). If $f(a) = f(b)$, then there is at least one number c in (a, b) such that $f'(c) = 0$.

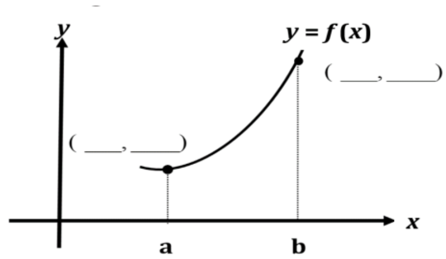
<u>Example 1</u>: Illustrating Rolle's Theorem
Determine if Rolle's Theorem applies to $f(x) = x^4 - 2x^2$ on the interval $[-2,2]$. State thoroughly the reasons why or why not the theorem applies. If the theorem does apply, find the value of c guaranteed by the theorem.
Confirm your results by sketching the graph



The Mean Value Theorem

If f is continuous on the closed interval $[a, b]$and differentiable on the open interval (a, b),then there exists a number c in (a, b)such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

Visualizing the Mean Value Theorem



1. In the figure to the right, label the indicated ordered pairs.

2. Label a segment whose length is $f(b) - f(a)$.

3. Label a segment whose length is $(b - a)$.

4. The quotient $\frac{f(b)-f(a)}{b-a}$ is the

______________________ of the segment joining the two points (________ , ________) and (________, ________).

Draw in this segment.

5. $\boldsymbol{f'(c)}$ gives the ________________________ of the line ___________________________ to the curve $f(x)$ at the point (________, ________).

6. On the graph above, locate the *c* value that is guaranteed by the Mean Value Theorem and draw a line tangent to the curve at *x* = *c*.

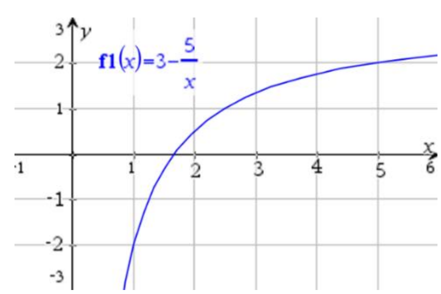
7. The two lines drawn on the graph of $f(x)$ are _____________________________ to each other.

8. Thus, the conclusion of the **Mean Value Theorem** can be restated as follows:

There will be two parallel lines, one through the points (________, ________) and (________, ________), and the other __________________________ to the curve $f(x)$ at the point (________, ________).

Example 2: Illustrating the Mean Value Theorem

Determine if The Mean Value Theorem applies to $f(x) = 3 - \frac{5}{x}$ on the interval [1,5]. State thoroughly the reasons why or why not the theorem applies. If the theorem does apply, find the value of *c* guaranteed by the theorem. Visually, what is the Mean Value Theorem trying to find? Draw it in.



Example 3: **Another Mean Value Theorem Problem**
Find the equation of the tangent line to the graph of $f(x) = 2x + sin\, x + 1$ on the interval $[0, \pi]$ at the point which is the solution to the Mean Value Theorem.

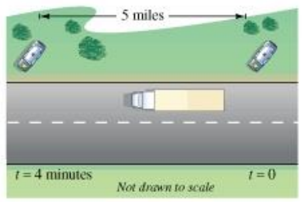
Example 4: **When the Mean Value Theorem Does Not Apply**
Explain precisely why we cannot apply the Mean Value Theorem to either of the three functions below on the provided intervals.

a. $f(x) = -|x - 3|$ on $[2,5]$

b. $g(x) = \frac{2}{x+2}$ on $[-3,1]$

c. $h(x) = x^{\frac{2}{3}}$ on $[-1,3]$

Example 5: **Real World Application of The Mean Value Theorem**
Two stationary police cars equipped with radar are 5 miles apart on a highway, as seen in the figure. As a semi-truck passes the first patrol car, its speed is clocked at 55 miles per hour. Four minutes later, when the truck passes the second patrol car, its speed is clocked at 50 miles per hour. Prove that the truck did or did not exceed the 55 mile per hour speed limit at some time between the two police cars.



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