Notes 5 – 2 Critical and Stationary points, Relative Extrema, Points of inflection

<u>CRITICAL POINTS</u>:

A critical point for a function f is any value of x in the **<u>domain</u>** of f at which

- $f'(x)=0$ ***(Stationary points)*** **or**
- f is continuous but not differentiable (i.e. 5.3 Cusps and points of vertical tangency)

<u>RELATIVE EXTREMA</u>

Transition points that separate regions where a graph is increasing from those where the graph is decreasing. (Algebra 2 definition)

They **will occur** at:

- points where the graph of f has a horizontal tangent
  - $f'(x)=0$
- points where f is not differentiable but continuous
  - $f'(x)$ is undefined and $f(x)$ is defined (i.e. 5.3 cusps and points of vertical tangency)

See graphs on page 281
The relative extrema of a function occur at <u>critical points</u> where $f'(x)$ changes signs. Not all critical points are extrema (i.e. 5.2 inflections points and 5.3 points of vertical tangency)

<u>FIRST DERIVATIVE TEST</u>- (Calculus Definition for Extrema)

Relative extrema must occur at **<u>critical points</u>** (implying $f'(x)=0$ or is undefined)

1. A relative maximum occurs at those critical points where the sign of the first derivative changes from positive to negative moving in the positive $x-$direction.

2. A relative minimum occurs at those critical points where the sign of the first derivative changes from negative to positive moving in the positive $x-$direction.

3. No relative extremum occurs at those points where the sign of the first derivative does not change moving in the positive $x-$direction.

<u>SECOND DERIVATIVE TEST</u> – (Calculus Definition for Extrema)

Suppose that f is twice differentiable at a **<u>stationary point</u>** x_0 (implying $f'(x_0)=0$)

- So if $f'(x_0)=0$ and $f''(x_0)>0$, implies $f'(x)$ is increasing and f has a relative minimum at x_0
- So if $f'(x_0)=0$ and $f''(x_0)<0$, implies $f'(x)$ is decreasing and f has a relative maximum at x_0
- So if $f'(x_0)=0$ and $f''(x_0)=0$, then the test is inconclusive. You may or may not have relative extrema at x_0.

NOTES ON DIFFERENTIABILITY:

- Differentiability at x_0 means the derivative is defined/exists
- When $f'(x_0)=0$ the 1st derivative EXISTS and the $f''(x_0)$ can be positive, negative or zero. When both are true $f(x)$ is twice differentiable at x_0.
- If f is **not** twice differentiable (meaning $f''(x_0)$ does not exist) at the critical point x_0 or if $f''(x_0)=0$, then you must rely on the first derivative test.

<u>"Points of Interest" (possible Inflection points)</u> come from the second derivative but you cannot call them critical points

- $f''(x)=0$ **or**
- points where $f''(x)$ is undefined

- Definition of Inflection point
    - Where $f(x)$ changes concavity (Algebra)
    - (Calculus) If $f(x)$ is **continuous** and $f''(x)$ changes signs, you have a point of inflection. Use this definition for your justification.

NOTES ON CONTINUITY:

- When $f'(x_0)$ is defined/exists (meaning $f'(x_0)$ is positive, negative or zero) then $f(x)$ **must** have been continuous
- When $f''(x_0)$ is defined/exists then $f'(x_0)$ must have been defined/exists
    - If $f'(x_0)$ exists, $f(x)$ must have been **continuous** which means x_0 is in the **DOMAIN** of $f(x)$.

SO, $f'(x_0)=0$ or $f''(x_0)=0$, implies $f(x)$ is **continuous** x_0 but not vice versa.