5.3 - 5.4 Summary

CONTINUITY

- When $f'(x_0)$ is defined/exists (meaning $f'(x_0)$ is positive, negative or zero) then $f(x)$ **must** have been continuous
- When $f''(x_0)$ is defined/exists then $f'(x_0)$ must have been defined/exists
  - If $f'(x_0)$ exists, $f(x)$ must have been **continuous** which means x_0 is in the **DOMAIN** of $f(x)$.
  - So, $f'(x_0) = 0$ or $f''(x_0) = 0$, implies $f(x)$ is **continuous** x_0 but not vice versa.

DIFFERENTIABILITY

- Differentiability at x_0 means the derivative is defined/exists
- When $f'(x_0) = 0$ the 1st derivative EXISTS and the $f''(x_0)$ can be positive, negative or zero. When both are true $f(x)$ is twice differentiable at x_0.

Guidelines for Finding Intervals on Which a Function is Increasing or Decreasing

Let *f* be **continuous** on the interval (a, b). To find the intervals on which *f* is increasing or decreasing, use the following steps.

1. Locate the critical numbers of *f* in (a, b), and use these numbers to determine your test intervals.

2. Determine the sign of $f'(x)$ by picking a "test value" in each of the intervals.

3. Use the Theorem for Increasing and Decreasing Functions to determine whether the function increases or decreases.

The guidelines above will also work if the interval (a, b) is replaced by an interval of the forms: $(-\infty, b)$, (a, ∞), or $(-\infty, \infty)$

THEOREM: Test for Increasing and Decreasing Functions

Let *f* be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

1. If $f'(x) > 0$ for all *x* in (a, b), then *f* is increasing on $[a, b]$.

2. If $f'(x) < 0$ for all *x* in (a, b), then *f* is decreasing on $[a, b]$.

Definition: A function is **strictly monotonic** on an interval if it is either increasing on the entire interval or decreasing on the entire interval.

First Derivative Test

THEOREM: The First Derivative Test

Let *c* be a critical number of the function *f* that is **continuous** on an open interval *I* containing *c*. If *f* is differentiable on the interval, except possibly at *c*, then $f(c)$ can be classified as follows.

1. If $f'(x)$ changes from negative to positive at c, then $f(c)$ is a *relative minimum* of *f.*

2. If $f'(x)$ changes from positive to negative at c, then $f(c)$ is a *relative maximum* of *f.*

3. If $f'(x)$ does not change its sign at c, then $f(c)$ is neither a *relative minimum* nor *relative maximum.*