Notes 5. 3 Rational Functions; Curves with Cusps and Vertical Tangent lines

1st Derivative:

If $f'(x)$ is undefined at x_0 and $f(x_0)$ is undefined, then x_0 cannot be a critical point, stationary point, point of vertical tangency or a cusp because x_0 is not in the **domain** of $f(x_0)$. (i.e. Rational Functions)

If $f'(x)$ is undefined at x_0 and $f(x_0)$ **IS** defined (i.e. curves with Cusps and Vertical Tangent lines), then x_0

1. Is a point of vertical tangency if there is NO sign change
2. Is a cusp if there IS a sign change

2nd Derivative:

Same as Polynomials 5.2 and if $f(x_0)$ is NOT defined (not continuous i.e. vertical asymptote), then x_0 cannot be a point of inflection.

Example 1: $f(x) = \frac{2x^2-8}{x^2-16}$ $f'(x) = \frac{-48x}{(x^2-16)^2}$ $f''(x) = \frac{48(16+3x^2)}{(x^2-16)^3}$

Find the x-intercepts.

Find the y-intercept.

Where is $f(x)$ undefined? **Explain what this tells you about** $f(x)$?

Where is $f(x)$ above/below the *x*-axis? Make a sign chart for $f(x)$

Are there any horizontal asymptotes? $\lim_{x\to\pm\infty} f(x)$

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Are there any oblique (slant) asymptotes?

What are the possible critical points of $f(x)$? **Explain what these tell you about** $f(x)$?

Make a sign chart for $f'(x)$ and label everything.

Make a sign chart for $f''(x)$ and label everything.

Sketch the graph of $f(x)$

Example 2: $f(x) = (x-4)^{\frac{2}{3}}$ $f'(x) = \frac{2}{3(x-4)^{\frac{1}{3}}}$ $f''(x) = \frac{-2}{9(x-4)^{\frac{4}{3}}}$

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