5.5 - 5.7 Summary

Finding Absolute Extrema on a Closed Interval

Guidelines for Finding Absolute Extrema on a Closed Interval

To find extrema of a continuous function f on a closed interval $[a, b]$, use the following steps.

1. Find the critical numbers of f in (a, b).
2. Evaluate f at each critical number in (a, b).
3. Evaluate f at each endpoint of $[a, b]$.
4. The least of these f values is the **absolute** minimum. The greatest is the **absolute** maximum.

These two steps are referred to as the Candidates Test

Note: The actual maximum or minimum value is a y value. Where the maximum or minimum occurs would be an x value.

Finding Absolute Extrema on an Open Interval **(usually not asked)**

Absolute Extrema on an **open interval** $(\boldsymbol{a}, \boldsymbol{b})$ may or may NOT have absolute extremum. On an open interval you must evaluate $\lim_{x \to a^+} f(x)$, relative extremum, and $\lim_{x \to b^-} f(x)$. ***Open*** endpoints **must** be evaluated but cannot be a location of Absolute Extrema.

Test for Concavity

Let f be a function whose second derivative exists on an open interval (a, b).

- If $f''(x) > 0$ for all x in (a, b), then the graph of f is concave upward on (a, b).
- If $f''(x) < 0$ for all x in (a, b), then the graph of f is concave downward on (a, b).

Definition of Inflection point

- If $f(x)$ is **continuous** and $f''(x)$ changes signs, you have a point of inflection. This is the Calculus definition you must use for your justification

"Points of Interest" (possible Inflection points) come from the second derivative but they are not called critical number ($f'(x) = 0$ or Undefined)

- $f''(x) = 0$ **or**
- $f''(x)$ is undefined, but $f(x)$ is defined, i.e. a point of vertical tangency
- **NOTE:** All points of vertical tangency are points of inflection but not vice versa

SECOND DERIVATIVE TEST

Suppose that f is twice differentiable at a **stationary point** x_0 (implying $f'(x_0) = 0$)

- If $f'(x_0) = 0$ and $f''(x_0) > 0$, implies $f'(x)$ is increasing and f has a relative minimum at x_0
- If $f'(x_0) = 0$ and $f''(x_0) < 0$, implies $f'(x)$ is decreasing and f has a relative maximum at x_0
- If $f'(x_0) = 0$ and $f''(x_0) = 0$ or $f''(x_0)$ does not exist) then this test is inconclusive. You may or may not have relative extrema at x_0. You must rely on the first derivative test