Name ______________________________

Skill Builder: Topic 5.5 – Using the Candidates Test to Determine Absolute (Global) Extrema

Find the absolute (global) maximum and absolute (global) minimum of the given function over the provided interval.

1.) $f(x) = 4x^2 - 4x + 1 [0,2]$

$f'(x) = 8x - 4$

$\underline{f'(x) = 0}$ $\qquad$ $\underline{f'(x) \text{ is undefined}}$

$8x - 4 = 0$ $\qquad$ $\varnothing$

$x = \frac{1}{2}$

Candidates: $0, \frac{1}{2}$, and 2

x	$f(x)$
0	$4(0)^2 - 4(0) + 1 = 1$
$\frac{1}{2}$	$4\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 1 = 0$
2	$4(2)^2 - 4(2) + 1 = 9$

The absolute maximum is 9 which occurs when $x = 2$.

The absolute minimum is 0 which occurs when $x = \frac{1}{2}$.

2.) $f(x) = 6x^3 - 6x^4 + 5 [-1,2]$

$f'(x) = 18x^2 - 24x^3$

$\underline{f'(x) = 0}$ $\qquad$ $\underline{f'(x) \text{ is undefined}}$

$18x^2 - 24x^3 = 0$ $\qquad$ $\varnothing$

$6x^2(3 - 4x) = 0$

$x = 0, \frac{3}{4}$

Candidates: $-1, 0, \frac{3}{4}$, and 2

x	$f(x)$
-1	$6(-1)^3 - 6(-1)^4 + 5 = -7$
0	$6(0)^3 - 6(0)^4 + 5 = 5$
$\frac{3}{4}$	$6\left(\frac{3}{4}\right)^3 - 6\left(\frac{3}{4}\right)^4 + 5 = \frac{721}{128}$
2	$6(2)^3 - 6(2)^4 + 5 = -43$

The absolute maximum is $\frac{721}{128}$ which occurs when $x = \frac{3}{4}$.

3.) $f(x) = (x^2 - 1)^{\frac{2}{3}} [-2,3]$

$f'(x) = \frac{2}{3}(x^2 - 1)^{-1/3}(2x) = \frac{4x}{3\sqrt[3]{x^2 - 1}}$

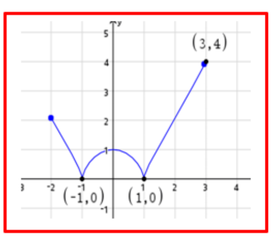
$\underline{f'(x) = 0}$ $\qquad$ $\underline{f'(x) \text{ is undefined}}$

$4x = 0$ $\qquad$ $3\sqrt[3]{x^2 - 1} = 0$

$x = 0$ $\qquad$ $x = -1, 1$

Candidates: $-1, 0$, and 1

x	$f(x)$
-2	$((-2)^2 - 1)^{\frac{2}{3}} = \sqrt[3]{9}$
-1	$((-1)^2 - 1)^{\frac{2}{3}} = 0$
0	$((0)^2 - 1)^{\frac{2}{3}} = 1$
1	$((1)^2 - 1)^{\frac{2}{3}} = 0$
3	$((3)^2 - 1)^{\frac{2}{3}} = 4$



The absolute maximum is 4 which occurs when $x = 3$.
The absolute minimum is 0 which occurs when $x = -1$ and $x = 1$.

4.) $f(x) = \sin x - \cos x [0, \pi]$

$f'(x) = \cos x + \sin x$

$\underline{f'(x) = 0}$ $\qquad$ $\underline{f'(x) \text{ is undefined}}$

$\cos x + \sin x = 0$ $\qquad$ $\varnothing$

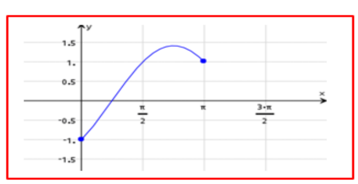
$\cos x = -\sin x$

$x = \frac{3\pi}{4}$ $\qquad$ Candidates: $0, \frac{3\pi}{4}$, and π

x	$f(x)$
0	$\sin(0) - \cos(0) = -1$
$\frac{3\pi}{4}$	$\sin\left(\frac{3\pi}{4}\right) - \cos\left(\frac{3\pi}{4}\right) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$
π	$\sin(\pi) - \cos(\pi) = 1$

The absolute maximum is $\sqrt{2}$ which occur occurs when $x = 3\pi/4$.

The absolute minimum is -1 which occurs when $x = 0$.

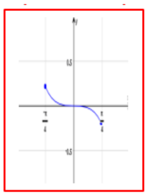


5.) $f(x) = x - \tan x \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

$f'(x) = 1 - \sec^2 x = 1 - \frac{1}{\cos^2 x} = \frac{\cos^2 x - 1}{\cos^2 x}$

$f'(x) = 0$	$f'(x)$ is undefined
$\cos^2 x - 1 = 0$	$\cos^2 x = 0$
$\cos^2 x = 1$	$\varnothing$ in $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$
$\cos x = 1 \quad \cos x = -1$	
$x = 0$	Candidates: $-\frac{\pi}{4}, 0,$ and $\frac{\pi}{4}$

x	$f(x)$
$-\frac{\pi}{4}$	$\left(-\frac{\pi}{4}\right) - \tan\left(-\frac{\pi}{4}\right) = -\frac{\pi}{4} + 1$
0	$(0) - \tan(0) = 0$
$\frac{\pi}{4}$	$\left(\frac{\pi}{4}\right) - \tan\left(\frac{\pi}{4}\right) = \frac{\pi}{4} - 1$



The absolute maximum is $-\frac{\pi}{4} + 1$ which occur occurs when $x = -\pi/4$.

The absolute minimum is $\frac{\pi}{4} - 1$ which occurs when $x = \pi/4$.

6.) What is the smallest slope of the function $y = x^3 - 3x^2 + 5x - 1$ on $\left[-\frac{1}{2}, 2\right]$?

We want to find the minimum of y'

$f'(x) = 3x^2 - 6x + 5$

$f''(x) = 6x - 6$

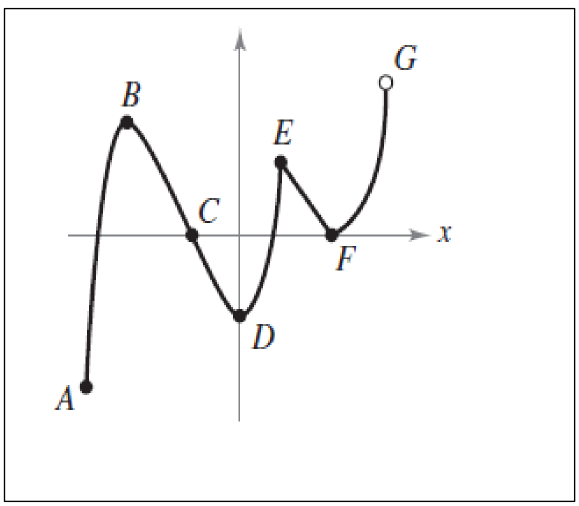
$f''(x) = 0$	$f''(x)$ is undefined
$6x - 6 = 0$	$\varnothing$
$x = 1$	

Candidates: $-\frac{1}{2}, 1,$ and 2

x	slope of $f(x)$ or $f'(x)$
$-\frac{1}{2}$	$3\left(-\frac{1}{2}\right)^2 - 6\left(-\frac{1}{2}\right) + 5 = \frac{3}{4} + 8 = \frac{35}{4}$
1	$3(1)^2 - 6(1) + 5 = 2$
2	$3(2)^2 - 6(2) + 5 = 5$

The minimum slope of $f(x)$ is 2 which occurs at $x = 1$.

7.) Determine whether each labeled point is an absolute maximum or minimum, a relative maximum or minimum or neither.



A	**absolute minimum**
B	**relative maximum**
C	**point of inflection**
D	**relative minimum**
E	**relative maximum**
F	**relative minimum**
G	**neither**

8.) Determine whether each statement is True or False. If a statement is false, explain why or give an example that shows it to be false.

a.) The maximum of a function that is continuous on a closed interval can occur at two different values in the interval.

True. (Many of the exercises found on this Skill Builder have demonstrated this.)

b.) If a function is continuous on a closed interval, then it must have a minimum on the interval.

True. (The is the conclusion of the Extreme Value Theorem.)

c.) If $x = c$ is a critical number of the function f, then it is also a critical number of the function $g(x) = f(x) + k$, where k is a constant.

True. (The constant value *k* will disappear after the first derivative is taken.)

d.) If $x = c$ is a critical number of the function f, then it is also a critical number of the function $g(x) = f(x - k)$, where k is a constant.

False. A horizontal shift of *k* units to the right will cause the critical value to move to the right as well.

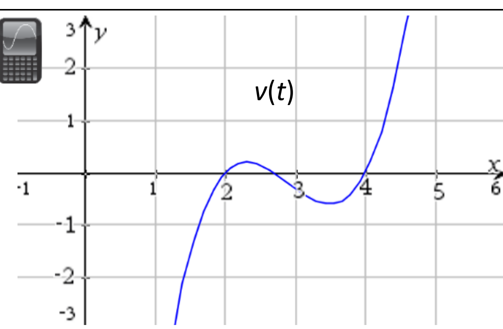
e.) Let the function f be differentiable on an interval I containing c. If f has a maximum value at $x = c$, then $-f$ has a minimum value at $x = c$.

True. (The negative sign will reflect the graph around the *x*-axis.)

f.) A quaratic function has a derivative defined by the cubic function $f'(x) = ax^3 + bx^2 + cx + d$ where $a \neq 0$ will always have exactly three critical numbers.

False. If the value(s) of *b* and/or *c* were equal to zero, there could be fewer unique solutions to $f'(x) = 0$. One such example would be $x^3 = 0$ which only has one unique solution, $x = 1$.

9.) A particle moves along the *x*-axis such that its position is $x(t) = 0.25t^4 - 2.916t^3 + 12.25t^2 - 22t + 21.6$ for $1 \leq t \leq 4$. A graph of its velocity is shown to the right. At what time does the particle reach its leftmost position? Where is the particle when it reaches its leftmost position?



$\frac{d}{dt}\left(0.25 \cdot t^4 - 2.916 \cdot t^3 + 12.25 \cdot t^2 - 22 \cdot t + 21.6\right)$ $t^3 - 8.748 \cdot t^2 + 24.5 \cdot t - 22.$

$\text{solve}\left(t^3 - 8.748 \cdot t^2 + 24.5 \cdot t - 22. = 0, t\right)$ $t = 1.99475$ or $t = 2.76619$ or $t = 3.98707$

$0.25 \cdot t^4 - 2.916 \cdot t^3 + 12.25 \cdot t^2 - 22 \cdot t + 21.6 | t = \{1, 1.99475, 2.76619, 3.98707, 4\}$

$\{9.184, 7.27198, 7.39491, 6.97579, 6.976\}$

The particle reaches its leftmost position at $t = 3.987$ at a location of approximately 6.975.

10.) If a particle moves along a straight line according to $s(t) = \frac{1}{12}t^4 - \frac{1}{3}t^3 - \frac{3}{2}t^2 + 4t - 7$, find

a.) the maximum and minimum velocity on $0 \le t \le 4$.

$v(t) = s'(t) = \frac{1}{3}t^3 - t^2 - 3t + 4$

$v'(t) = t^2 - 2t - 3$

$\underline{v'(t) = 0}$ $\qquad$ $\underline{v'(t) \text{ is undefined}}$

$t^2 - 2t - 3 = 0$ $\qquad$ $\varnothing$

$(t-3)(t+1) = 0$

$t = 3, \cancel{-1}$

Candidates: 0,1,3, and 4

t	$v(t)$
0	$\frac{1}{3}(0)^3 - (0)^2 - 3(0) + 4 = 4$
1	$\frac{1}{3}(1)^3 - (1)^2 - 3(1) + 4 = \frac{1}{3}$
3	$\frac{1}{3}(3)^3 - (3)^2 - 3(3) + 4 = -5$
4	$\frac{1}{3}(4)^3 - (4)^2 - 3(4) + 4 = \frac{64}{3} - 24 = -\frac{8}{3}$

The minimum velocity is -5 which occurs when $t = 3$.
The maximum velocity is 4 which occurs when $t = 0$.

b.) the maximum and minimum acceleration on $0 \le t \le 4$.

$a(t) = v'(t) = t^2 - 2t - 3$

$a'(t) = 2t - 2$

$\underline{a'(t) = 0}$ $\qquad$ $\underline{a'(t) \text{ is undefined}}$

$2t - 2 = 0$ $\qquad$ $\varnothing$

$t = 1$

Candidates: 0,1, and 4

t	$v(t)$
0	$(0)^2 - 2(0) - 3 = -3$
1	$(1)^2 - 2(1) - 3 = -4$
4	$4^2 - 2(4) - 3 = 5$

The minimum acceleration is -4 which occurs when $t = 1$.
The maximum acceleration is 5 which occurs when $t = 4$.