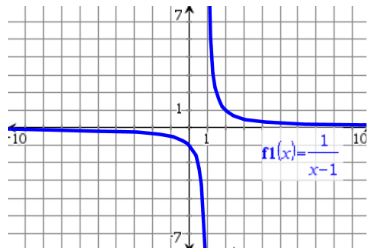
FUN		
2	Topic: 5.6 Topic: 5.7	Determining Concavity of Functions Over Their Domains Using the Second Derivative Test to Determine Relative (Local) Extrema
Learning Objective FUN-4.A: Justify conclusions about the behavior of a function based on the behavior of its derivatives.		

Concavity

Consider the following function:

$f(x) = \frac{1}{x-1}$



Note: $f(x)$ is concave downward on $(-\infty, 1)$ and concave upward on $(1, \infty)$

<u>Definition of Concavity</u>
Let *f* be differentiable on an open interval *I*. The graph of *f* is **concave upward** on *I* if *f* ' is increasing on the interval and **concave downward** on *I* if *f* ' is decreasing on the interval.

<u>Test for Concavity</u>
Let f be a function whose second derivative exists on an open interval (a,b).

1. If $f''(x) > 0$ for all *x* in (a,b), then the graph of f is concave upward on (a,b).

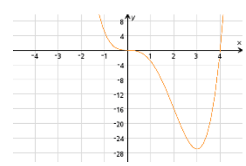
2. If $f''(x) < 0$ for all *x* in (a,b), then the graph of f is concave downward on (a,b).

<u>Definition of a Point of Inflection</u>
A point of inflection is an ordered pair where a graph changes concavity.

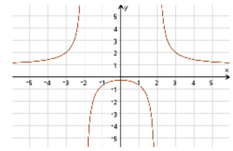
<u>Points of Inflection Theorem</u>
If $(c, f(c))$ is a point of inflection of the graph of *f*, then either $f''(x) = 0$ or *f* is not differentiable at *x* = *c*.

<u>Example 1</u>: Determine the open intervals on which each graph is concave upward or downward and state any points of inflection. Justify your answer.

a. $f(x) = x^4 - 4x^3$



b. $f(x) = \frac{6}{x^2+3}$



The Second Derivative Test

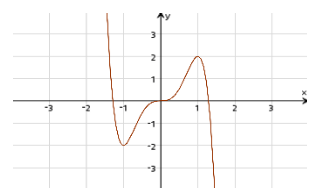
Now, we will investigate another way to find the maximum and minimum values of a function.

The Second Derivative Test

Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.

1. If $f''(c) > 0$, then $f(c)$ is a relative minimum.
2. If $f''(c) < 0$, then $f(c)$ is a relative maximum.

Example 2: Find the relative extrema for $f(x) = -3x^5 + 5x^3$ using the Second Derivative Test.



Example 3: Given the values below for x, $f(x)$, $f'(x)$, along with the fact that $f'(x)$ has only 2 zeros on the interval $(-3, 10)$ and $f''(x)$ has only 2 zeros on the interval $(-3, 10)$, answer each of the following.

x	-3	-1	1	3	5	7	10
$f(x)$	-7	1	-1	-4	3	2	-1
$f'(x)$	1	0	-1	0	2	undefined	3
$f''(x)$	-2	-1	0	2	3	0	5

a. Identify all x-values where f has a relative minimum. Justify using the First Derivative Test.

b. Identify all x-values where f has a relative maximum. Justify using the Second Derivative Test.

c. Identify all x-values where f has a point of inflection. Justify.

d. What is the equation of the tangent to the curve $y = f(x)$ at $x = 5$?