FUN	Notes Unit 6		
1	Topic: 6.2	Approximating Areas with Riemann Sums Using left and right endpoints	
Learning Objective LIM-5.A: Approximate a definite integral using geometric and numerical methods.			

Although there are many possible techniques to approximate the area under a curve, we will use rectangles (Riemann Sums) or trapezoids in AP Calculus

The two big ideas in Calculus are the <u>tangent line problem</u> and the <u>area problem</u>.

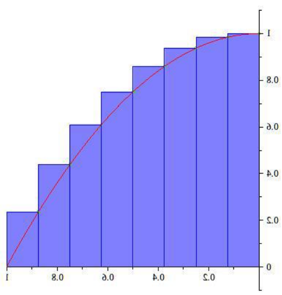
- In the tangent line problem, we saw how the limit process could be applied to the slope of a line to find the slope of a general curve.
- A second classic problem in Calculus is in finding the area of a plane region that is bounded by the graphs of functions. In this case, the limit process is applied to the area of a rectangle to find the area of a general region.

A basic overview of "areas as limits",

- In the "limit of rectangles" approach, we take the area under a curve by approximating a collection of <u>inscribed</u> rectangles, <u>circumscribed</u> rectangles, or a more accurate approach of using midpoints. We are interested in finding the area of a region bounded by the x-axis which means no portion of its graph on the interval is below the x-axis. These methods are commonly known as Riemann Sums. When the number of rectangles is increased without limit, we get the actual area. Known as Integration.

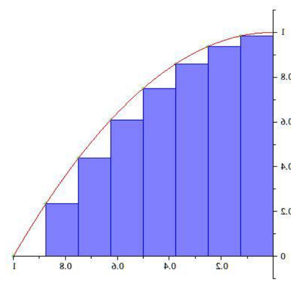
When the function is increasing:

Circumscribed Rectangles
(Upper Sum)
Overestimates



Right end point approximation

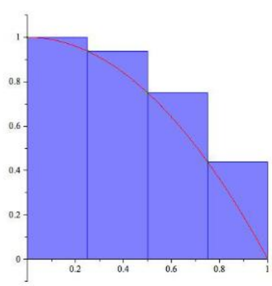
Inscribed Rectangles
(Lower Sum)
Underestimates



Left end point approximation

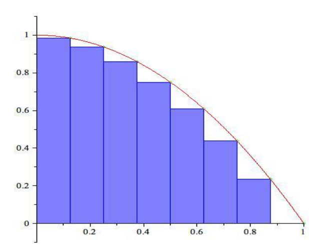
When the function is decreasing:

Circumscribed Rectangle
(Upper Sum)
Overestimates



Left end point approximation

Inscribed Rectangles
(Lower Sum)
Underestimates



Right end point approximation

<u>Approximating the Area Under a Curve Using Rectangles</u>

When using **rectangles** to approximate the area under a curve, we must decide a few things in advance.

1. How many rectangles will we use?
   (We will call these **subintervals**.)
2. How will we determine the height of each rectangle?
   (Our options will be **left endpoint**, **midpoint**, or **right endpoint**.)

<u>Method for approximating area using Riemann Sums:</u>

1. Draw a rough sketch of the function over the interval

2. Use the formula $\Delta x = \frac{b-a}{n}$ to determine each subinterval, the width of each rectangle

3. Compute the areas of each rectangle using the formula $\Delta x \cdot f(x^*)$

4. Find the summation of the approximated areas $A = \sum_{k=1}^{n} \Delta x \cdot f(x^*)$

MATHEMATICS & HISTORY

GEORG FRIEDRICH BERNHARD RIEMANN
(1826-1866)
German mathematician Riemann did his most famous work in the area of non-Euclidean geometry, differential equations, and number theory. It was Riemann's results in physics and mathematics that formed the structure on which Einstein's Theory of Relativity is based.

Ex.1 Approximate the area under the curve of $f(x) = 2x - 3$ in the interval $[2, 6]$.

a.) Use Geometry

b) Divide the interval into 4 subintervals of equal length and compute the lower sum (inscribed rectangles). Is that on over or underestimate? Explain.

c) Divide the interval into 4 subintervals of equal length and compute the upper sum (circumscribed rectangles). Is that on over or underestimate? Explain.

Ex.2 Approximate the area under the curve of $f(x) = -x^2 + 25$ in the interval $[0, 4]$.

a.) Use Geometry

b) Divide the interval into 4 subintervals of equal length and compute the upper sum.
Is that on over or underestimate? Explain.

What do you think will happen with our approximated area if we were to use more rectangles?

AP Exam Tip

On the AP Exam, it is very important that you show a **sum** of **products** whenever you are constructing an approximation of an area using either Riemann sums or trapezoidal sums. In other words, don't perform any arithmetic calculation in your first step. The readers want to see the structure of your approximation method.