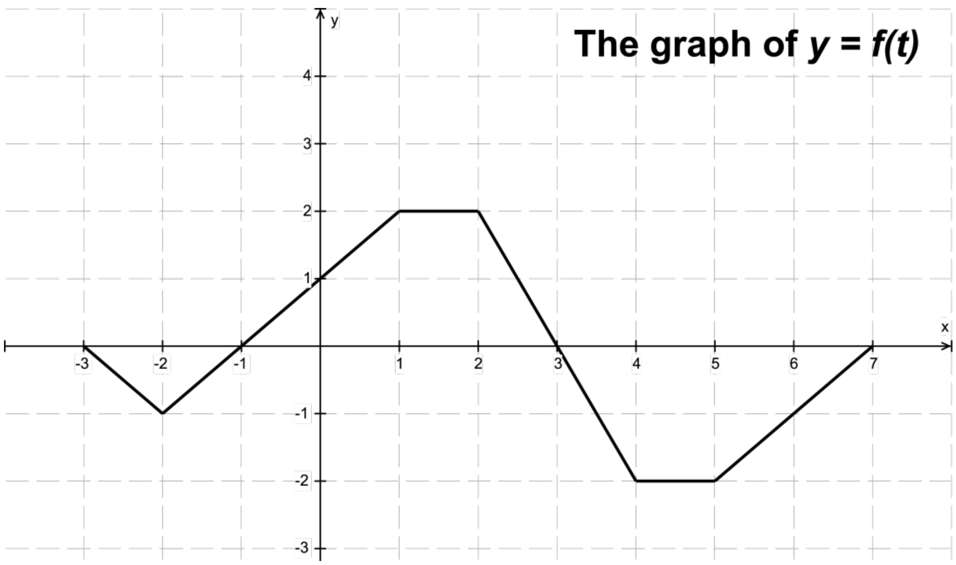
Determining Values of $g(x)=\int_{-1}^{x} f(t)dt$ from the Graph of $y=f(t)$

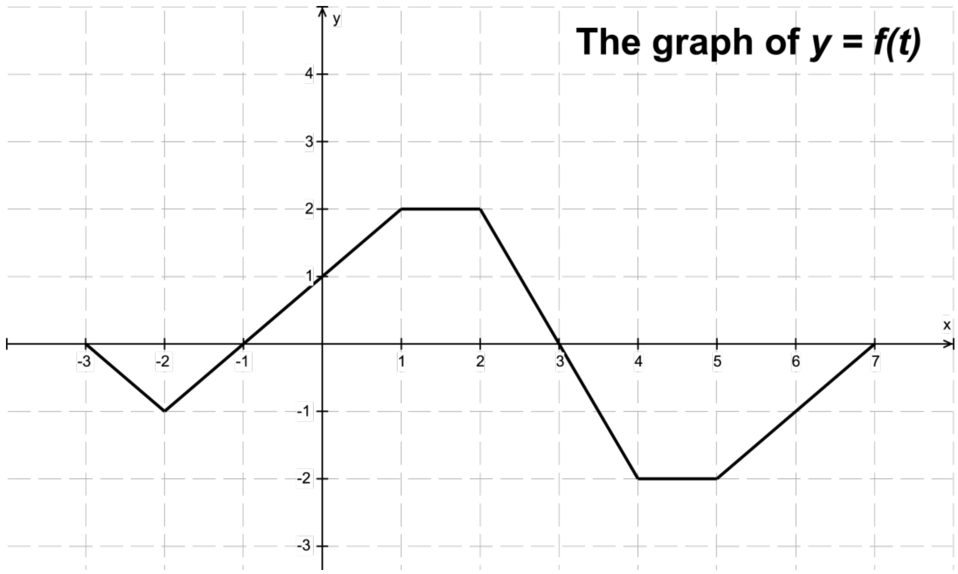
Let $g(x)=\int_{-1}^{x} f(t)\,dt$. The graph of $y=f(t)$ is given below.



Fill in the values of the table. Note that the lower limit is -1.

x	$g(x)$
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	
6	
7	

Using the graph below, plot the points from the table to help you graph $y = g(x)$. Be sure to indicate all relative extrema and points of inflection.



Notes:

Use your graph as an aide to answer the following questions.

1. On what interval(s) is the graph of $y = g(x)$ increasing?

2. On what interval(s) is the graph of $y = g(x)$ decreasing?

3. On what interval(s) is the graph of $y = g(x)$ concave up?

4. On what interval(s) is the graph of $y = g(x)$ concave down?

5. How can you use the graph of *f* to determine where $y = g(x)$ has a local minimum?

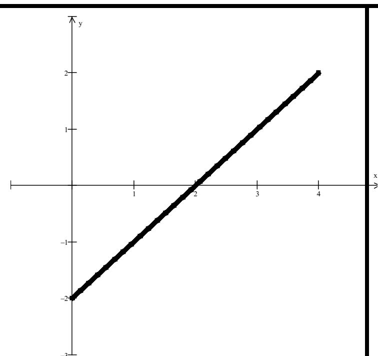
6. How can you use the graph of *f* to determine where $y = g(x)$ has a local maximum?

7. How can you use the graph of *f* to determine where $y = g(x)$ has a point of inflection?

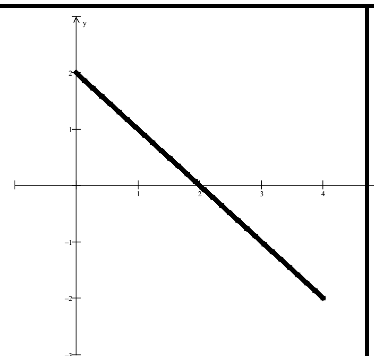
Selecting Graphs of *f* Given a Description of *g*

You are presented with 12 graphs of $y = f(t)$. Each graph is continuous on the interval [0, 4] and differentiable on the interval (0, 4). Let $g(x) = \int_0^x f(t)dt$. For the questions that follow, list ***all*** the graphs that would make each statement true. Then state what condition or feature is present in the graphs of *f*. Recall the relationship between the functions *f* and *g*. The first question is done for you.

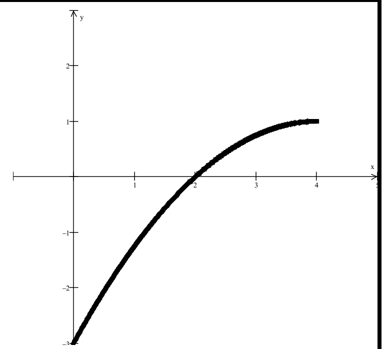
A



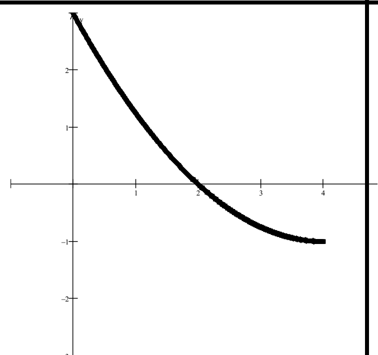
B



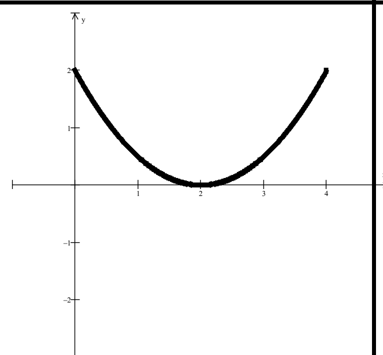
C



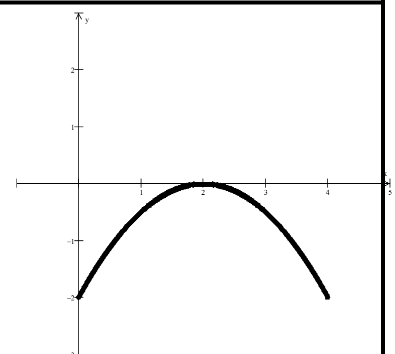
D



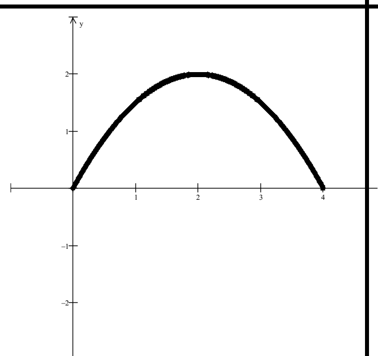
E



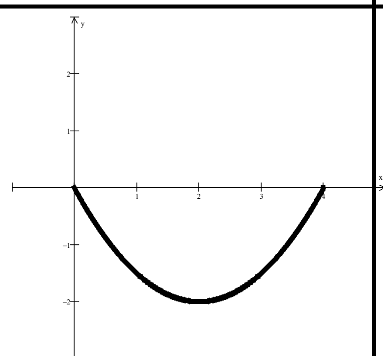
F



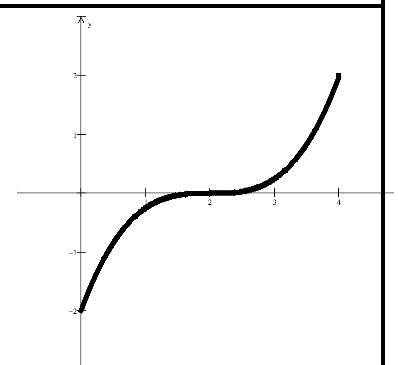
G



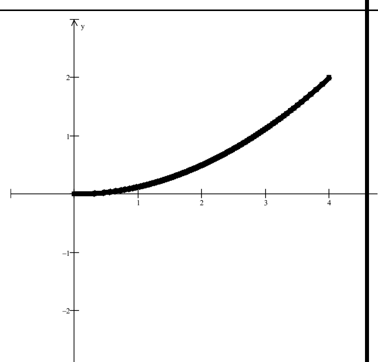
H



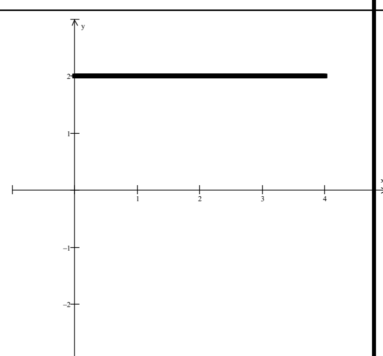
I



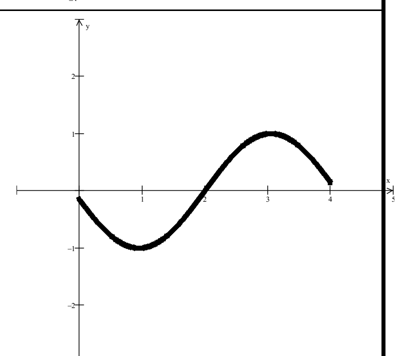
J



K



L



1. **Example:** The graph of $y = g(x)$ is increasing on (0, 4).
   - Which graphs, if any, would make this statement true?
     Answer: E, G, J, K
   - If applicable: What condition or feature is present in the graphs of *f*?
     Answer: These graphs of *f* are all positive on (0, 4).

2. The graph of $y = g(x)$ is decreasing on (0, 4).
   - Which graphs, if any, would make this statement true?
   - If applicable: What condition or feature is present in the graphs of *f*?

3. The graph of $y = g(x)$ is constant on (0, 4).
   - Which graphs, if any, would make this statement true?
   - If applicable: What condition or feature is present in the graphs of *f*?

4. The graph of $y = g(x)$ is concave up on (0, 4).
   - Which graphs, if any, would make this statement true?
   - If applicable: What condition or feature is present in the graphs of *f*?

5. The graph of $y = g(x)$ is concave down on (0, 4).
   - Which graphs, if any, would make this statement true?
   - If applicable: What condition or feature is present in the graphs of *f*?

6. The graph of y has a local maximum at $x = 2$.
   - Which graphs, if any, would make this statement true?

- If applicable: What condition or feature is present in the graphs of *f*?

7. The graph of $y = g(x)$ has a local minimum at $x = 2$.

- Which graphs, if any, would make this statement true?
- If applicable: What condition or feature is present in the graphs of *f*?

8. The graph of $y = g(x)$ has a point of inflection at $x = 2$.

- Which graphs, if any, would make this statement true?
- If applicable: What condition or feature is present in the graphs of *f*?

9. The graph of $y = g(x)$ is increasing on (0, 2) and decreasing on (2, 4).

- Which graphs, if any, would make this statement true?
- If applicable: What condition or feature is present in the graphs of *f*?

10. The graph of $y = g(x)$ is concave up on (0, 2) and concave down on (2, 4).

- Which graphs, if any, would make this statement true?
- If applicable: What condition or feature is present in the graphs of *f*?

11. The graph of $y = g(x)$ has no local extrema on (0, 4).

- Which graphs, if any, would make this statement true?
- If applicable: What condition or feature is present in the graphs of *f*?

12. The graph of $y = g(x)$ has no points of inflection on (0, 4).

- Which graphs, if any, would make this statement true?

- If applicable: What condition or feature is present in the graphs of *f*?

Check your understanding

Determine whether each statement is true or false. It the statement is false, explain why it is false or edit the statement to make it a true statement.

If $g(x) = \int_0^x f(t)dt$, then $g'(x) = f(x)$.

If $g(x) = \int_0^x f(t)dt$, and *f* is increasing on (0, 2), then *g* is increasing on (0, 2).

If $g(x) = \int_0^x f(t)dt$, and *f* is positive on (0, 4), then *g* is concave up on (0, 4).

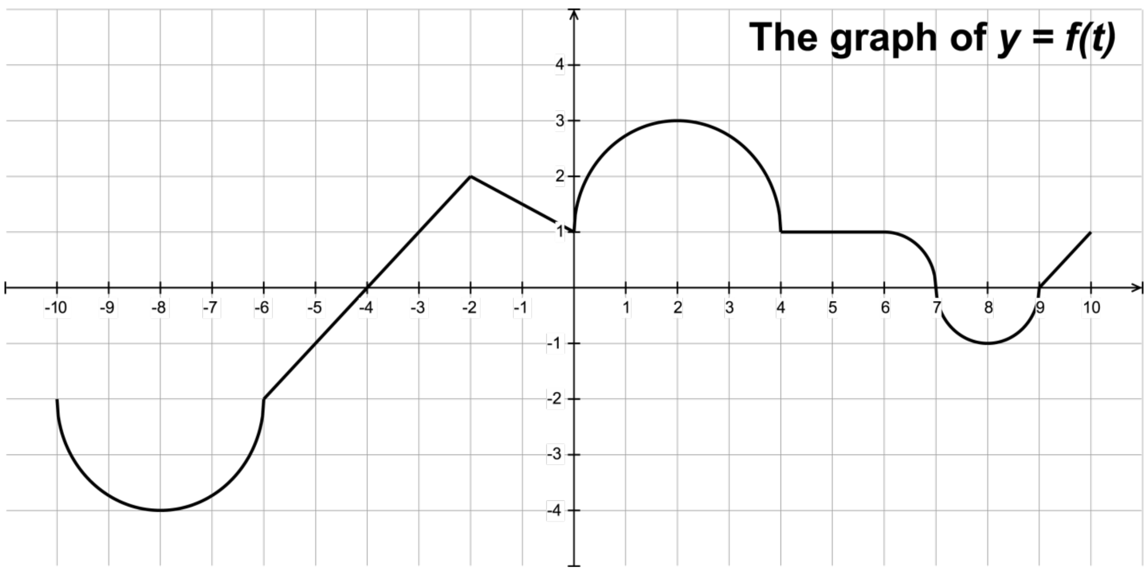
If $g(x) = \int_0^x f(t)dt$, and *f* changes sign from positive to negative at *x* = 3, then *g* has a point of inflection at *x* = 3.

If $g(x) = \int_0^x f(t)dt$, and *f* has a local minimum at *x* = 5, then *g* has a point of inflection at *x* = 5.

If $g(x) = \int_0^x f(t)dt$, and *g* is negative on (0, 4), then *f* must be decreasing on (0, 4).

Apply Your Understanding of Functions Defined by Integrals

Part I: Let $g(x) = \int_{-4}^{x} f(t)\,dt$. The graph of $y = f(t)$ is given below.



Fill in each statement with the interval(s) and reason. The first one is done for you.

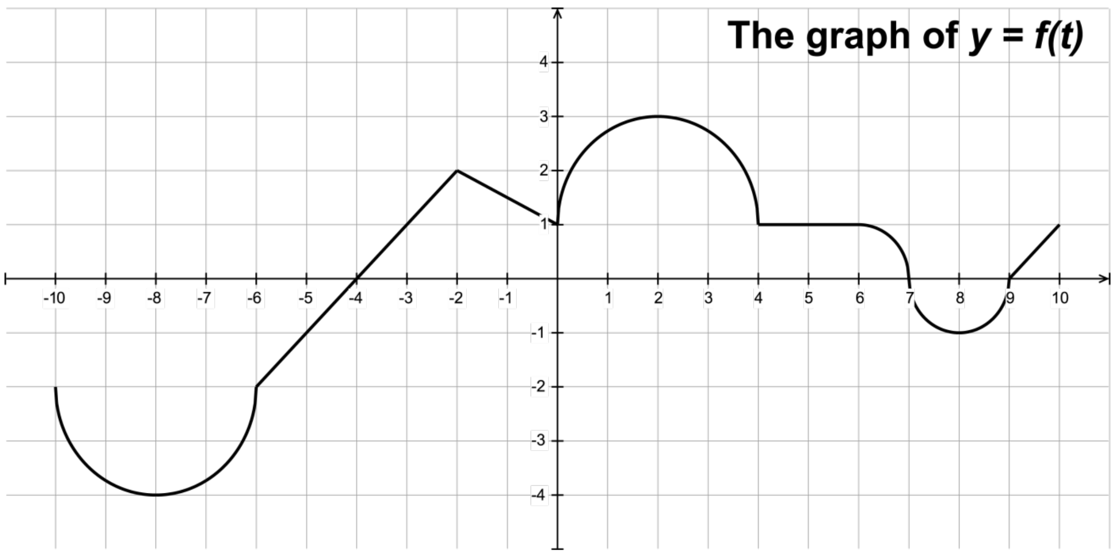
1. The graph of $y = g(x)$ is increasing on (-4, 7) and (9, 10) because this is where $g'(x) = f(x)$ is positive.

2. The graph of $y = g(x)$ is decreasing on ________ and ________ because ______________ ____.

3. The graph of $y = g(x)$ is concave up on ___________________________________ because ____.

4. The graph of $y = g(x)$ is concave down on _________________________________ because ____.

Part II: Let $g(x) = \int_0^x f(t)dt$. The graph of $y = f(t)$ is given above where *f* is continuous on the interval [0, 12]. Finish each statement and provide a reason for your answer. Remember that $f = g'$.



1. $(x) = \int_0^x f(t)dt$ is increasing on the following intervals:

This is because:

2. $g(x) = \int_0^x f(t)dt$ is decreasing on the following intervals:

This is because:

3. $g(x) = \int_0^x f(t)dt$ is concave up on the following intervals:

This is because:

4. $g(x) = \int_0^x f(t)dt$ is concave down on the following intervals:

This is because:

5. $g(x) = \int_0^x f(t)dt$ has a local maximum at the following *x* values:

This is because:

6. $g(x) = \int_0^x f(t)dt$ has a local minimum at the following *x* values:

This is because:

7. $g(x) = \int_0^x f(t)dt$ has a point of inflection at the following *x* values:

This is because:

Check your understanding

Complete each statement choosing from the terms: increasing, decreasing, positive, negative, zero, changes sign, has a local extrema, has a point of inflection, concave up, or concave down. (Note: Not all terms will be used.)

$g(x)=\int_0^x f(t)dt$ is increasing whenever the graph of *f* is:

$g(x)=\int_0^x f(t)dt$ is decreasing whenever the graph of *f* is:

$g(x)=\int_0^x f(t)dt$ is concave up whenever the graph of *f* is:

$g(x)=\int_0^x f(t)dt$ is concave down whenever the graph of *f* is:

$g(x)=\int_0^x f(t)dt$ has a local extrema whenever the graph of *f*:

$g(x)=\int_0^x f(t)dt$ has a point of inflection whenever the graph of *f*: