Topic: 7.1 Modeling Situations with Differential Equations

Learning Objective FUN-7.A: Interpret the verbal statements of problems as differential equations involving a derivative expression

This unit contains one of the most important applications of calculus – ***differential equations***.

A **differential equation** in *x* and *y* is an equation that involves *x, y,* and the derivatives of *y.*

A **solution to a differential equation** is a function *y* that satisfies the differential equation when the function and its derivatives are substituted into the equation. A solution to a differential equation could be one function **(singular solution)** or a set of functions **(general solution)**.

The **order** of a differential is the highest derivative it contains

Notation

Examples of Differential Equations	$y''+2y'=3y$	$f''(x)+2f'(x)=3f(x)$	$\frac{d^2y}{dx^2}+2\frac{dy}{dx}=3y$

Example 1: A particle moves along a straight line. Its velocity, v, is inversely proportional to the square of the distance, s, it has traveled. Which equation describes this relationship?

(A) $v(t)=\frac{k}{t^2}$ **(B)** $v(t)=\frac{k}{s^2}$ **(C)** $\frac{dv}{dt}=\frac{k}{t^2}$ **(D)** $\frac{dv}{dt}=\frac{k}{s^2}$

Example 2: A puppy gains weight, w, at a rate approximately inversely proportional to its age, t, in months. Write an equation that describes this relationship.

Example 3: The learning rate for a skill, S, is proportional to the difference between the maximum potential for learning that skill, M, and the amount of that skill already learned, L. Write an equation that describes this relationship.

Example 4: An IV drip administers medication to a patient's bloodstream at a rate of 3 cubic centimeters per hour. At the same time, the patient's organs remove the medication from the patient's bloodstream at a rate proportional to the current volume V of medication in the bloodstream.Which equation describes this relationship?

(A) $\frac{dV}{dt}=3-kV$ **(B)** $\frac{dV}{dt}=-3kV$ **(C)** $\frac{dV}{dt}=k-3v$ **(D)** $\frac{dV}{dt}=3x-V$

Example 5: A chemical is diluted out of a tank by pumping pure water into the tank and pumping the existing solution out of it, so the volume at any time t is $20+2t$.
The amount z of chemical in the tank decreases at a rate proportional to z and inversely proportional to the volume of solution in the tank.
Which equation describes this relationship?

(A) $\frac{dz}{dt}=-\frac{kz}{20+2t}$ **(B)** $\frac{dz}{dt}=kz-\frac{1}{20+2t}$

(C) $\frac{dz}{dt}=k(20+2t)-\frac{1}{z}$ **(D)** $\frac{dz}{dt}=-\frac{k(20+2t)}{z}$

Topic: 7.2 Verifying Solutions for Differential Equations

Learning Objective FUN-7.B: Verify solutions to differential equations

Recall, a function $y = f(x)$ is called a **solution** of a differential equation if the equation is satisfied when *y* and its derivatives are replaced by $f(x)$ and its derivatives. When at all possible, we strive to solve a differential equation by writing *y* all by itself (y is a **solution**).

Example 1: Verifying Solutions to Differential Equations.

Determine whether the given function is a solution of the differential equation $f''(x) - f(x) = 0$.

a. $f(x) = \sin x$

b. $f(x) = 4e^{-x}$

Example 2:

Determine whether the given function is a solution of the differential equation $\frac{dy}{dx} = \frac{4y}{x}$.

a. $y = 4x$

b. $y = x^4$

Example 3:

For what values of *k*, if any, is $y = e^{2x} + ke^{-3x}$ a solution to the differential equation $4y - y'' = 10e^{-3x}$?

(A) -2 **(B)** $\frac{10}{3}$ **(C)** 10 **(D)** There is no such value of *k*