Function Operations

Addition	$(f+g)(x) = f(x) + g(x)$
Subtraction	$(f-g)(x) = f(x) - g(x)$
Multiplication	$(f \cdot g)(x) = f(x) \cdot g(x)$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

Example: Let $f(x) = 5x + 12$ and $g(x) = 3x - 8$

Find: $(f+g)(x)$

$$(f+g)(x) = (5x+12) + (3x-8)$$
$$= 5x + 3x + 12 + (-8)$$
$$= 8x + 4$$

Find: $(f-g)(x)$

$$(f-g)(x) = (5x+12) - (3x-8)$$
$$= (5x+12) + (-3x+8)$$
$$= 5x + (-3x) + 12 + 8$$
$$= 2x + 20$$

Example 1: Let $f(x) = x^2 + 6x$ and $g(x) = -2x + 10$

Find: $(f+g)(x)$

Find: $(f-g)(x)$

Example: Let $f(x) = x^2 - 1$ and $g(x) = x - 1$

Find: $(f \cdot g)(x)$ $(f \cdot g)(x) = (x^2 - 1) \cdot (x - 1)$

$= x^3 - x^2 - x + 1$

Find: $\left(\frac{f}{g}\right)(x)$ $\left(\frac{f}{g}\right)(x) = \frac{x^2-1}{x-1}$ $x \neq 1$

$= \frac{(x+1)(x-1)}{x-1}$ $x \neq 1$

$= x + 1$ $x \neq 1$

Example 2: Let $f(x) = 2x^2 + 8x$ and $g(x) = x + 4$

Find: $(f \cdot g)(x)$

Find: $\left(\frac{f}{g}\right)(x)$

Composition of Functions

$g \circ f(x) = g\big(f(x)\big)$

1. Evaluate the inner function $f(x)$ first.
2. Then use your answer as the input of the outer function $g(x)$.

Example: Let $f(x) = x^2$ and $g(x) = 2x + 3$

Find: $g \circ f(4)$

$$f(4) = 4^2 = 16$$

$$g(16) = 2(16) + 3 = 35$$

$$g \circ f(4) = 35$$

Find: $f \circ g(4)$

$$g(4) = 2(4) + 3 = 11$$

$$f(11) = 11^2 = 121$$

$$f \circ g(4) = 121$$

Example 3: Let $f(x) = 2x - 1$ and $g(x) = x + 10$

Find: $f \circ g(5)$

Find: $f \circ g(-2)$

Find: $g \circ g(8)$

Example: Let $f(x) = x^2$ and $g(x) = 2x + 3$

Find: $g \circ f(x)$

$f(x) = x^2$

$g(x^2) = 2(x^2) + 3 = 2x^2 + 3$

Find: $f \circ g(x)$

$g(x) = 2(x) + 3 = 2x + 3$

$f(2x + 3) = (2x + 3)^2 = 4x^2 + 12x + 9$

Example 4: Let $f(x) = 2x - 1$ and $g(x) = x + 10$

Find: $f \circ g(x)$

Find: $g \circ f(x)$