BC CALCULUS - PARAMETRIC EQUATIONS

Given the parametric equations $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$ in which $f(t)$ and $g(t)$ have continuous first derivatives with respect to t, then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad (dx/dt \neq 0)$.

The second derivative is given by $\frac{d^2y}{dx^2} = \frac{\frac{d\left(dy/dx\right)}{dt}}{dx/dt}$.

Horizontal Tangent Lines occur when $\frac{dy}{dt} = 0 \ \ and \ \ \frac{dx}{dt} \neq 0.$

Vertical Tangent Lines occur when $\frac{dx}{dt} = 0 \ \ and \ \ \frac{dy}{dt} \neq 0.$

Singular points occur where $\frac{dy}{dt} = 0 \ \ and \ \ \frac{dx}{dt} = 0.$

Problems:

1. Use the parametric curve $x = \frac{t}{2}, \ y = t^2 + 1$ for the following.
    a. Sketch the curve.
    b. Find the slope of the line tangent to the parametric curve at $t = -1$.
    c. Find the equation of the line tangent to the curve at $t = 1$.
    d. Find $\frac{d^2y}{dx^2}$ at $t = 3$.
    e. Find any points where horizontal tangents, vertical tangents, or singular points occur.

2. Use the parametric curve $x = 3cost, \ y = 4sint$ for the following.
    a. Sketch the curve.
    b. Find the slope of the line tangent to the parametric curve at $t = \frac{\pi}{4}$.
    c. Find the equation of the line tangent to the curve at $t = \frac{7\pi}{4}$.
    d. Find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{6}$.
    e. Find any points where horizontal tangents, vertical tangents, or singular points occur.

3. Use the parametric curve $x = t^3, \ y = t^2$ for the following.
    a. Sketch the curve over the interval $-2\pi \leq t \leq 2\pi$
    b. Find the slope of the line tangent to the parametric curve at $t = 2$.
    c. Find the equation of the line tangent to the curve at $t = -1.5$.
    d. Find $\frac{d^2y}{dx^2}$ at $t = 2$.
    e. Find any points where horizontal tangents, vertical tangents, or singular points occur.