9.1 + 9.2

① b) $x(t) = \frac{t}{2}$ $\quad y(t) = t^2 + 1$

$\frac{dx}{dt} = \frac{1}{2}$ $\quad \frac{dy}{dx} = 2t$

$$\frac{dy}{dx} = \frac{2t}{1/2} = 4t$$

$$\left.\frac{dy}{dx}\right|_{t=-1} = 4(-1) = \boxed{-4} = \text{slope @ } t=-1$$

c) $$\left.\frac{dy}{dx}\right|_{t=1} = 4(1) = 4$$

$x(1) = \frac{1}{2}$; $y(1) = 2$

$$\boxed{y - 2 = 4\left(x - \tfrac{1}{2}\right)}$$

d) $$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}[4t]}{dx/dt} = \frac{4}{1/2} = \boxed{8}$$

* 2nd derivative for ALL t. *

means curve is concave up

(21) b) $x(t) = 3\cos t$; $y(t) = 4\sin t$

$\frac{dx}{dt} = -3\sin t$; $\frac{dy}{dt} = 4\cos t$

$$\frac{dy}{dx} = \frac{4\cos t}{-3\sin t} = -\frac{4}{3}\cot(t)$$

$$\left.\frac{dy}{dx}\right|_{t=\pi/4} = -\frac{4}{3}\cot(\pi/4) = -\frac{4}{3}\cdot 1 = \boxed{-\frac{4}{3}}$$

c) $$\left.\frac{dy}{dx}\right|_{t=\frac{7\pi}{4}} = -\frac{4}{3}\cot\left(\frac{7\pi}{4}\right) = -\frac{4}{3}\cdot -1 = \frac{4}{3} = \text{slope } @t=\frac{7\pi}{4}$$

$$x\left(\frac{7\pi}{4}\right) = 3\cos\left(\frac{7\pi}{4}\right) = 3\cdot\frac{\sqrt{2}}{2} = \frac{3\sqrt{2}}{2}$$

$$y\left(\frac{7\pi}{4}\right) = 4\sin\left(\frac{7\pi}{4}\right) = 4\cdot -\frac{\sqrt{2}}{2} = -2\sqrt{2}$$

$$\boxed{y + 2\sqrt{2} = \frac{4}{3}\left(x - \frac{3\sqrt{2}}{2}\right)}$$

d) $$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[-\frac{4}{3}\cot(t)\right]}{dx/dt} = \frac{+\frac{4}{3}\csc^2(t)}{-3\sin(t)} = \frac{-4}{9}\csc^3(t)$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\pi/6} = -\frac{4}{9}\csc^3\left(\frac{\pi}{6}\right) = -\frac{4}{9}(2)^3 = \boxed{\frac{-32}{9}}$$

(3) $x(t) = t^3$; $y(t) = t^2$

b) $\frac{dx}{dt} = 3t^2$; $\frac{dy}{dt} = 2t$

$$\frac{dy}{dx} = \frac{2t}{3t^2} = \frac{2}{3t} \; ; \; \left.\frac{dy}{dx}\right|_{t=2} = \frac{2}{3(2)} = \boxed{\frac{1}{3}}$$

c) $\left.\frac{dy}{dx}\right|_{t=1.5} = \frac{2}{3(1.5)} = \frac{2}{-4.5} = \frac{2}{-9/2} = \frac{-4}{9}$ = slope at $t = -3/2$

$x(-1.5) = \left(\frac{-3}{2}\right)^3 = \frac{-27}{8}$; $y(-1.5) = \left(\frac{-3}{2}\right)^2 = \frac{9}{4}$

$$\boxed{y - \frac{9}{4} = \frac{-4}{9}\left(x + \frac{27}{8}\right)}$$

d) $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[\frac{2}{3t}\right]}{dx/dt} = \frac{\frac{-2 \cdot 3}{(3t)^2}}{3t^2}$ (I used reciprocal rule)

$$= \frac{-6}{9t^2 \cdot 3t^2} = \frac{-6}{27t^4} = \frac{-2}{9t^4}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=2} = \frac{-2}{9(2)^4} = \frac{-1}{9 \cdot 2^3} = \boxed{\frac{-1}{72}}$$