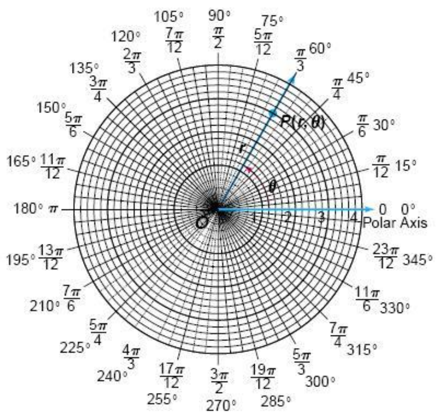
FUN	AP CALCULUS BC	
2	Topic: 9.7A	Defining Polar Coordinates and Differentiating in Polar Form

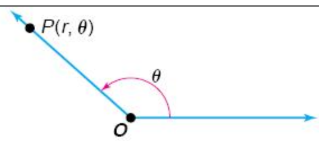
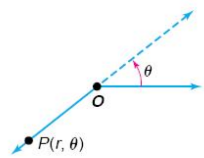
Learning Objective FUN-3.G: Calculate derivatives of functions written in polar coordinates.

Polar Coordinates



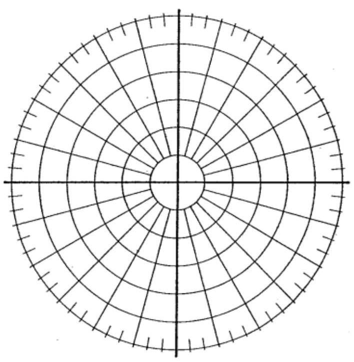
In a polar coordinate system, a fixed point O is called the **pole** or origin. The **polar axis** is usually a horizontal ray directed toward the right from the pole. The location of a point P in the polar coordinate system can be identified by polar coordinates in the form (r, θ). If a ray is drawn from the pole through P, the distance from the pole to point P is $|r|$. The measure of the angle formed by $\overrightarrow{OP}$ and the polar axis is θ. The angle can be measured in degrees or radians.

It is important to consider both positive and negative values of r.

Suppose $r > 0$. Then θ is the measure of any angle in standard position that has $\overrightarrow{OP}$ as its terminal side.	Suppose $r < 0$. Then θ is the measure of any angle that has the ray opposite $\overrightarrow{OP}$ as its terminal side.
	

Example 1: Graph each point.

a. $P\left(-1.5, \frac{7\pi}{6}\right)$



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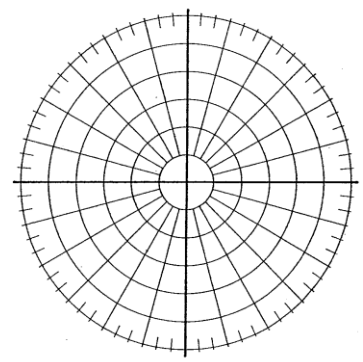
b. $Q\left(2, -\frac{\pi}{3}\right)$



Coordinate Conversions

Converting Polar Coordinates to Rectangular Coordinates	The rectangular coordinates (x, y) of a point named by the polar coordinates (r, θ) can be found by using the following formulas. $x = r\cos\theta$ $y = r\sin\theta$

Converting Rectangular Coordinates to Polar Coordinates	The polar coordinates (r, θ) of a point named by the rectangular coordinates (x, y) can be found by the following formulas. $r = \sqrt{x^2 + y^2}$ $\theta = \text{Arctan}\,\frac{y}{x}$, when $x > 0$ $\theta = \text{Arctan}\,\frac{y}{x} + \pi$, when $x < 0$

Example 2: Convert the following polar points to rectangular points.

a.) $\left(2\sqrt{3}, \frac{\pi}{6}\right)$	**b.)** $\left(8, \frac{5\pi}{4}\right)$

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Example 3: Convert the following rectangular points to polar points.

a.) $(3, -3)$	**b.)** $\left(-4, 4\sqrt{3}\right)$

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Equation Conversions

Since points can be written in both rectangular and polar format, we can also write equations with both formats. Graphs that are linear are best written in rectangular format, while graphs that have curves are best written in polar form. Non-functions with loops are very difficult to write in rectangular form and usually need to be written as a piecewise function while in many cases, are very easily written in polar form. To convert from one form to another, we use the same relationships in the boxes above.

Example 4: Convert the following rectangular equations to polar equations.

a.) $y = 6$	**b.)** $4x - 2y - 1 = 0$
c. $y = 4x^2$	

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Example 5: Convert the following polar equations to rectangular equations.

a.) $r = -\csc\theta$

b.) $r = \frac{2}{\sin\theta + 3\cos\theta}$

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c.) $r = \frac{3}{1 + 2\cos\theta}$

To convert a graph in polar form to parametric form, we use the fact that $x = r\cos\theta$ and $y = r\sin\theta$.

Example 6: Convert the following polar equations to parametric equations.

a.) $r = 4$

b.) $r = 4\sin\theta$

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c.) $r = \frac{4}{\cos\theta}$

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Polar Graphs

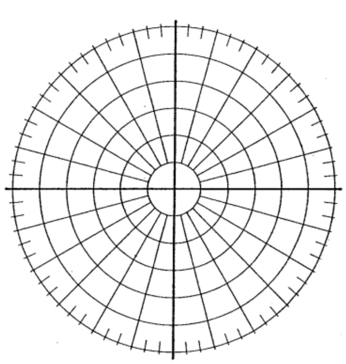
Example 7: **Graphing Polar Equations**

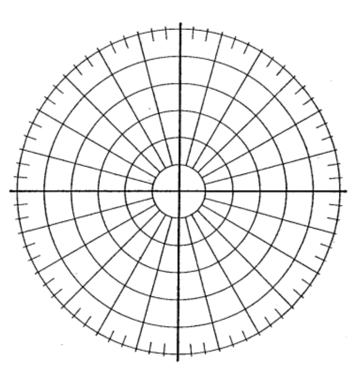
Graph each polar equation on the given polar coordinate planes.

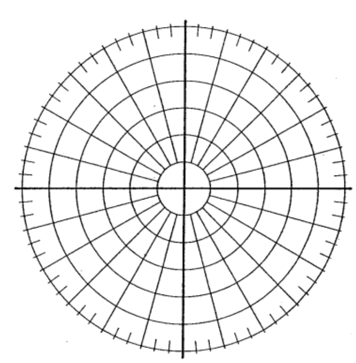
a.) $r = 2$

b.) $\theta = \frac{\pi}{3}$

c.) $r = \sec\theta$







Example 8: Graphing Polar Equations
Sketch the graph of the polar equation $r = 3\,cos\,3\,\theta$ on the given polar coordinate plane.

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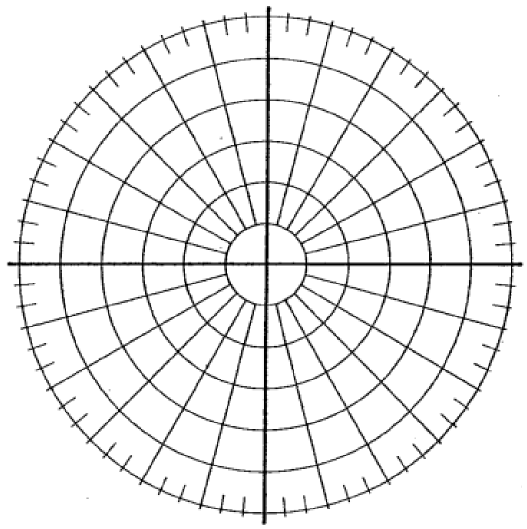
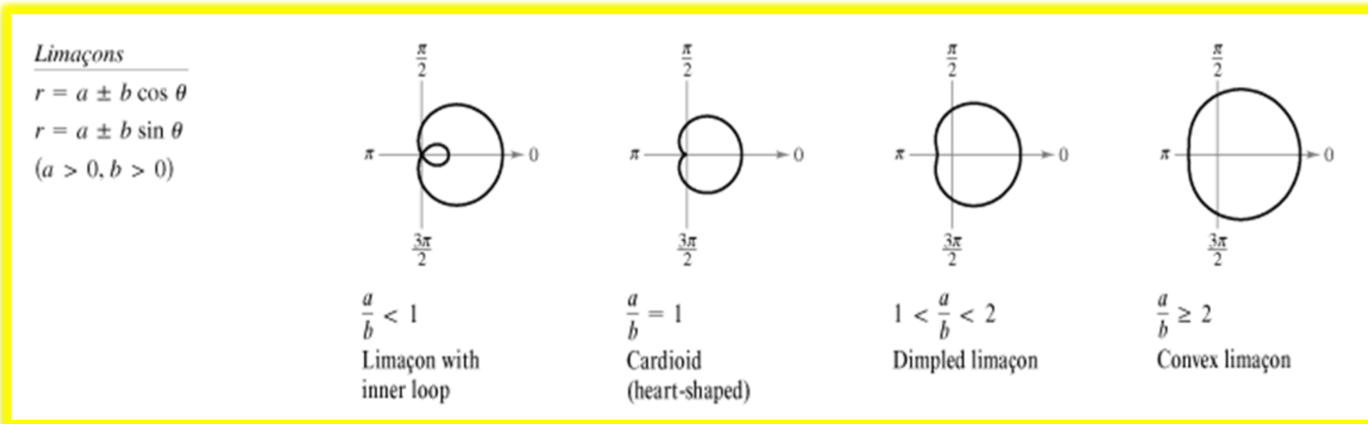
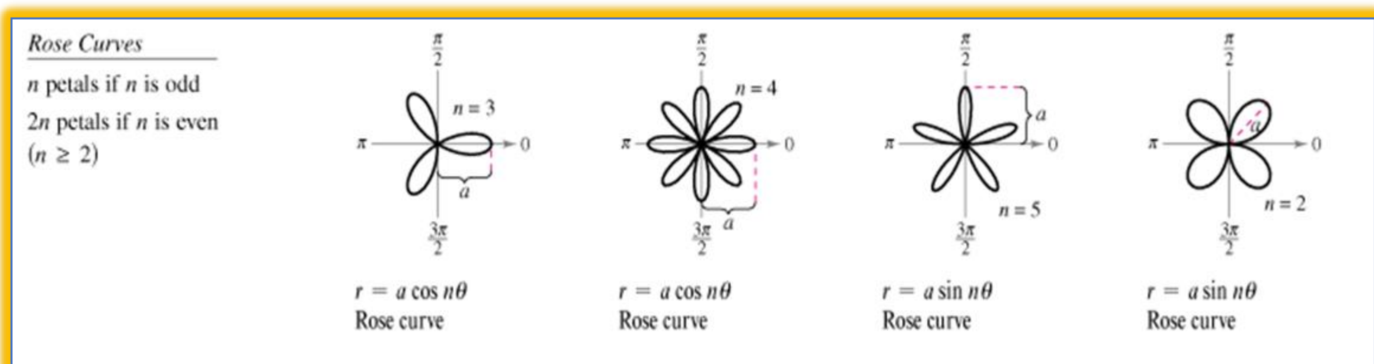


Special Polar Graphs

Limaçons



Rose Curves



Circles and Lemniscates