FUN	AP CALCULUS BC	
2	Topic: 9.7B	Differentiating in Polar Form

Learning Objective FUN-3.G: Calculate derivatives of functions written in polar coordinates.

Slope and Tangent Lines

To find the slope of a tangent line to a polar graph, consider a differentiable function defined by $r = f(\theta)$. To find the slope in polar form, use the parametric equations
$x = r\cos\theta = f(\theta)\cos\theta$ and $y = r\sin\theta = f(\theta)\sin\theta$
Using the parametric form of dy/dxgiven in the previous section, you have

> **THEOREM 9.7 SLOPE IN POLAR FORM**
>
> If f is a differentiable function of Θ, then the *slope* of the tangent line to the graph of $r = f(\theta)$ at the point (r, θ) is
> $$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

Using the formula above, we make these similar observations (as we did with parametric equations):

- To find horizontal tangents to polar graphs, set $\frac{dy}{d\theta} = 0$ provided that $\frac{dx}{d\theta} \neq 0$.
- To find vertical tangents to polar graphs, set $\frac{dx}{d\theta} = 0$ provided that $\frac{dy}{d\theta} \neq 0$.
- If $\frac{dx}{d\theta}$ and $\frac{dy}{d\theta}$ are simultaneously zero, we can make no conclusions about tangent lines.

Example 1: Heart Examination

Consider the cardioid described by the polar equation $r = 1 + \sin\theta$.

a.) Find the slope of the tangent line to the cardioid when $\theta = \frac{\pi}{3}$.

b.) Find the points on the cardioid where the tangent line is horizontal or vertical.

Example 2: Particle Motion of a Polar Curve

A polar curve r is given by $r = \theta + \cos\theta$, where $0 \leq \theta \leq 2\pi$. A particle is traveling along the polar curve r so that its position at time t is $(x(t), y(t))$and such that $\frac{d\theta}{dt} = \frac{1}{2}$. Find $\frac{dx}{dt}$ at the instant $\theta = \frac{5\pi}{6}$, and interpret the meaning of your answer in the context of the problem.