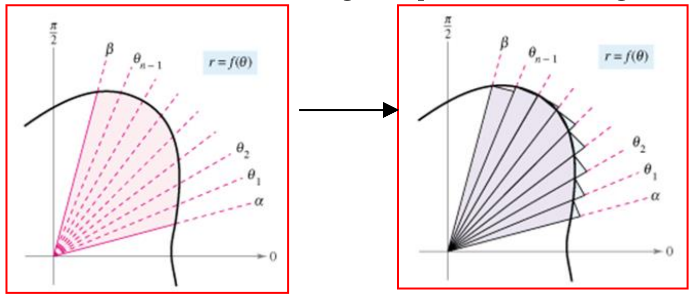
CHA	AP CALCULUS BC	
3	Topic: 9.8	Finding the Area of a Polar Region or the Area Bounded by a Single Polar Curve
Learning Objective CHA-5.D: Calculate areas of regions defined by polar curves by using definite integrals.		

Area of a Polar Region

The area of a sector of a circle is $A = \frac{1}{2}\theta r^2$.

The development of a formula for the area of a polar region parallels that for the area of the region on the rectangular coordinate system. The primary difference is that in polar, we use **sectors of circles** instead of rectangles to partition off our region.



Consider the function given by $r = f(\theta)$, where f is continuous and nonnegative on the interval given by $\alpha \le \theta \le \beta$. The region bounded by f and the radial lines $\theta = \alpha$ and $\theta = \beta$is what we wish to find.
If we partition $[\alpha, \beta]$ into n subintervals where $\alpha = \theta_0 < \theta_1 < \theta_2 < \cdots < \theta_{n-1} < \theta_n = \beta$
And let the radius of the ith sector be $f(\theta_i)$ and the central angle of the ith sector be $\frac{\beta-\alpha}{n} = \Delta\theta$,
Then our area can be approximated as $A \approx \sum_{i=1}^{n} \left(\frac{1}{2}\right) \Delta\theta [f(\theta_i)]^2$.
Taking the limit as $n \to \infty$produces

$$A = \lim_{n\to\infty} \frac{1}{2} \sum_{i=1}^{n} \left[f(\theta_i) \right]^2 \Delta\theta$$

$$= \frac{1}{2} \int_{\alpha}^{\beta} \left[f(\theta) \right]^2 d\theta$$

THEOREM 9.8: AREA IN POLAR COORDINATES

If f is continuous and nonnegative on the interval $[\alpha, \beta]$, $0 < \beta - \alpha < 2\pi$, then the area of the region bounded by the graph of $r = f(\theta)$ between the radial lines $\theta = \alpha$and $\theta = \beta$is given by

$$A = \frac{1}{2} \int_{\alpha}^{\beta} [f(\theta)]^2 d\theta$$

Example 1: Finding the Area of a Polar Region.
Find the area of one petal of the rose curve given by $r = 3\,cos\,3\,\theta$.

Example 2: Finding the Area Bounded by a Single Curve

Find the area of the region lying between the inner and outer loops of the limaçon $r = 1 - 2\,sin\,\theta$.

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