Algebra 2 – Absolute Value Equations and Inequalities **Name _____**

Absolute Value – the distance from zero on a number line

(absolute value is always a positive number)

Examples:

$|23| =$ $|-15| =$ $-|8| =$ $-|-125| =$

Solving Absolute Value Equations

$|x| = 2$ $x = -2$ or $x = +2$

$|15 - 3x| = 6$ $15 - 3x = -6$ or $15 - 3x = +6$

$4 - 2|x + 9| = -5$ $x + 9 = -4.5$ or $x + 9 = +4.5$

$-2|x + 9| = -9$

$|x + 9| = 4.5$

NOTE $|x + 3| = -5$ has no solution, since $|x + 3|$ cannot be negative

Extraneous Solution – a solution derived from an original equation that is not a solution of the original equation.

Checking for extraneous solutions:

Example: $|3x - 4| = -4x - 1$

$3x - 4 = -(-4x - 1)$ $\qquad$ $3x - 4 = -4x - 1$

$3x - 4 = 4x + 1$ $\qquad$ $7x = 3$

$-5 = x$ $\qquad$ $x = \frac{3}{7}$

Checking solutions:

$|3x - 4| = -4x - 1$ $\qquad$ $|3x - 4| = -4x - 1$

$|3(-5) - 4| = -4(-5) - 1$ $\qquad$ $\left|3\left(\frac{3}{7}\right) - 4\right| = -4\left(\frac{3}{7}\right) - 1$

Absolute Value Inequalities

$\lvert x\rvert \geq k$	$\lvert x\rvert \leq k$
$x \leq -k$ or $x \geq k$	$-k \leq x \leq k$

Solving Absolute Value Inequalities

Example: $|2x - 5| > 3$

$2x - 5 < -3$ or $2x - 5 > 3$

Example: $\frac{1}{3}|4 - 2x| \leq 2$

$|4 - 2x| \leq 6$

$-6 \leq 4 - 2x \leq 6$

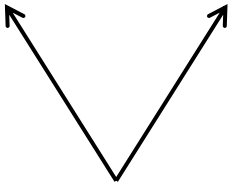
$-10 \leq -2x \leq 2$

$5 \geq x \geq -1$

$-1 \leq x \leq 5$

Absolute Value Functions and Graphs

Absolute Value Function $f(x) = |mx + b| + c$

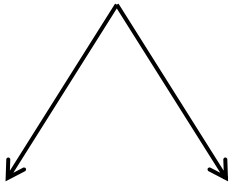


Vertex

$\left(-\frac{b}{m}, c\right)$

Note: graphs when $f(x) = -|x|$

The graph opens downward

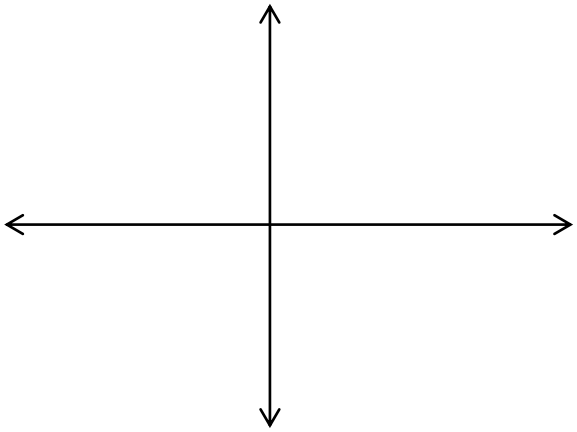


Graphing an absolute value function

Example: graph $y = |2x - 1|$ using a table of values.

Begin with $x = -\frac{b}{m} = -\frac{-1}{2} = \frac{1}{2}$

x	*y*
-1	
0	
$\mathbf{\frac{1}{2}}$	
1	
2	



Example: Using a graphing calculator to graph an absolute value function.

$f(x) = |x - 1| - 1$

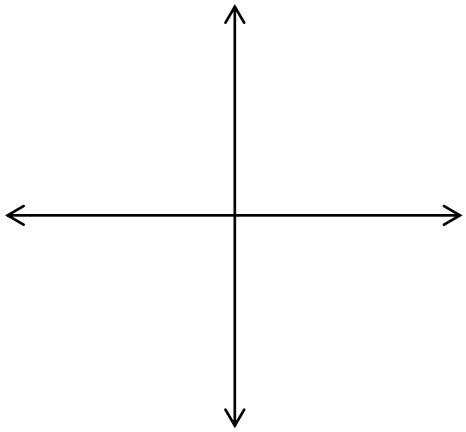
Vertex:

$x = -\frac{b}{m} = -\frac{-1}{1} = 1$

$f(1) = |1 - 1| - 1 = -1$

$(1, -1)$ is the vertex

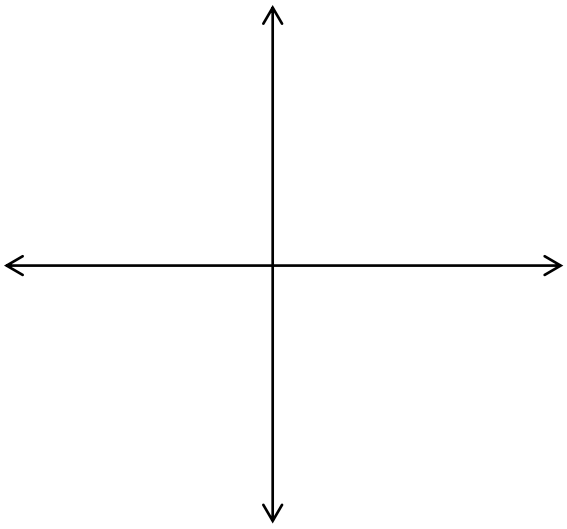
x	*y*
1	**−1**



Example: graph $y = -|2x + 8| - 5$

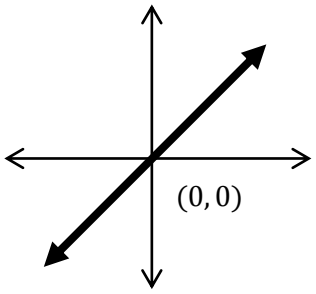
Vertex?

x	*y*

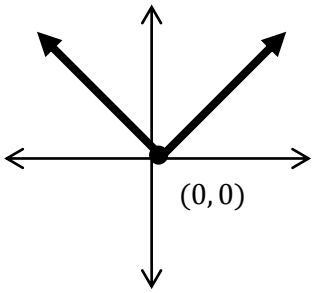


Translations of Linear Graphs and Absolute Value Graphs

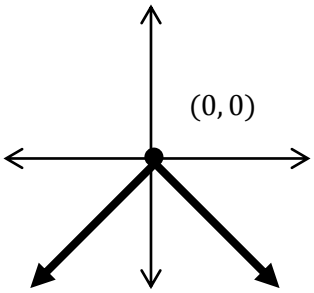
Here are the graphs of three parents graphs:



$y = x$



$y = |x|$



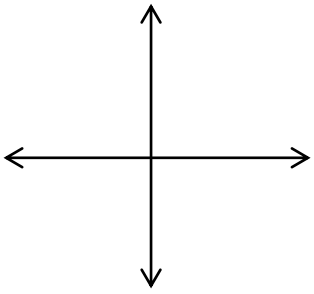
$y = -|x|$

<u>Vertical translations</u>

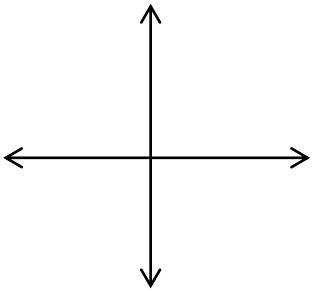
$y = x + k$ $y = |x| + k$ $y = -|x| + k$

will move the graph *k* units up if *k* is positive, and *k* units down if *k* is negative.

Ex: graph $y = |x| - 3$



graph $y = x + 2$

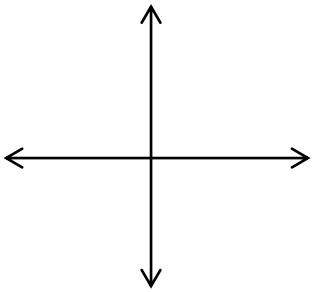


Horizontal translations

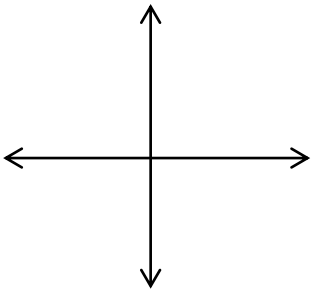
$y = (x - h)$ $y = |x - h|$ $y = -|x - h|$

will move the graph *h* units to the right if *h* is positive, and *h* units to the left if *h* is negative.

Ex: graph $y = -|x - 1|$



graph $y = |x + 5|$



If we combine a vertical and a horizontal translation the result is a diagonal transformation.

Ex: Describe the diagonal transformation, then graph $y = |x - 2| + 3$