

Final Exam Review

For each problem, find the area of the region enclosed by the curves.

1) $y = -x^2 + 4x$, $y = -x$,
 $x = 1$, $x = 5$

A) $\int_1^5 (-x^2 + 4x + x) dx$
 $= \frac{59}{3} \approx 19.667$

B) $\int_1^5 (-x^2 + 4x + x) dx$
 $= \frac{56}{3} \approx 18.667$

C) $\int_1^5 (-x^2 + 4x + x) dx$
 $= \frac{50}{3} \approx 16.667$

D) $\int_1^5 (-x^2 + 4x + x) dx$
 $= \frac{109}{6} \approx 18.167$

For each problem, find the area under the curve over the given interval. Set up, but do not evaluate the integral.

2) $y = -\frac{2}{x}$; $[-3, -1]$

A) $\int_{-3}^{-1} -4 \cdot -\frac{2}{x} dx$

B) $\int_{-3}^{-1} -\frac{2}{x} dx$

C) $\int_{-3}^{-1} -2 \cdot -\frac{2}{x} dx$

D) $\int_{-3}^{-1} 3 \cdot -\frac{2}{x} dx$

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the the x -axis.

3) $y = x^2, y = 0, x = 2$

$$\begin{aligned} \text{A)} \quad & \pi \int_0^2 (x^2)^2 dx \\ &= \frac{32}{5}\pi \approx 20.106 \end{aligned}$$

$$\begin{aligned} \text{B)} \quad & \pi \int_0^2 (x^2)^2 dx \\ &= \frac{22}{5}\pi \approx 13.823 \end{aligned}$$

$$\begin{aligned} \text{C)} \quad & \pi \int_0^2 (x^2)^2 dx \\ &= \frac{69}{10}\pi \approx 21.677 \end{aligned}$$

$$\begin{aligned} \text{D)} \quad & \pi \int_0^2 (x^2)^2 dx \\ &= \frac{91}{15}\pi \approx 19.059 \end{aligned}$$

4) $y = -x^2 + 4, y = 0, x = 0, x = 2$

$$\begin{aligned} \text{A)} \quad & \pi \int_0^2 (-x^2 + 4)^2 dx \\ &= \frac{271}{15}\pi \approx 56.758 \end{aligned}$$

$$\begin{aligned} \text{B)} \quad & \pi \int_0^2 (-x^2 + 4)^2 dx \\ &= \frac{527}{30}\pi \approx 55.187 \end{aligned}$$

$$\begin{aligned} \text{C)} \quad & \pi \int_0^2 (-x^2 + 4)^2 dx \\ &= \frac{512}{15}\pi \approx 107.233 \end{aligned}$$

$$\begin{aligned} \text{D)} \quad & \pi \int_0^2 (-x^2 + 4)^2 dx \\ &= \frac{256}{15}\pi \approx 53.617 \end{aligned}$$

For each problem, approximate the area under the curve over the given interval using 4 left endpoint rectangles.

5) $y = x^2 + 2x + 2; [-4, 0]$

- | | |
|--------------------------|-------|
| A) 20 | B) 18 |
| C) $\frac{37}{2} = 18.5$ | D) 54 |

For each problem, find the area of the region enclosed by the curves.

6) $y = -x^2 + 2, y = -x + 3, x = -1, x = 3$

$$\begin{aligned} \text{A)} \quad & \int_{-1}^3 (-x + 3 - (-x^2 + 2)) dx \\ &= \frac{28}{3} \approx 9.333 \end{aligned}$$

$$\begin{aligned} \text{B)} \quad & \int_{-1}^3 (-x + 3 - (-x^2 + 2)) dx \\ &= 9 \end{aligned}$$

$$\begin{aligned} \text{C)} \quad & \int_{-1}^3 (-x + 3 - (-x^2 + 2)) dx \\ &= \frac{25}{3} \approx 8.333 \end{aligned}$$

$$\begin{aligned} \text{D)} \quad & \int_{-1}^3 (-x + 3 - (-x^2 + 2)) dx \\ &= \frac{56}{3} \approx 18.667 \end{aligned}$$

Evaluate each sum.

7) $\sum_{k=1}^n (2k^2 + 1)$

A) $\frac{2n^3}{3} + n^2 + \frac{4n}{3}$

B) $\frac{4n^3}{3} + 2n^2 + \frac{8n}{3}$

C) $\frac{8n^3}{3} + 4n^2 + \frac{16n}{3}$

D) $\frac{n^3}{6} + \frac{n^2}{4} + \frac{n}{3}$

For each problem, approximate the area under the curve over the given interval using 4 left endpoint rectangles.

9) $y = x^2 + 2x + 3$; $[-4, 0]$

A) $\frac{43}{2} = 21.5$

B) 66

C) 44

D) 22

For each problem, use a left-hand Riemann sum to approximate the integral based off of the values in the table.

11) $\int_0^9 f(x) dx$

x	0	1	4	8	9
f(x)	8	6	7	5	3

A) 66

B) 54

C) 50

D) 59

For each problem, find the area under the curve over the given interval. Set up, but do not evaluate the integral.

8) $y = \frac{x^2}{2} - x + \frac{3}{2}$; $[3, 5]$

A) $\int_3^5 -4\left(\frac{x^2}{2} - x + \frac{3}{2}\right) dx$

B) $\int_3^5 3\left(\frac{x^2}{2} - x + \frac{3}{2}\right) dx$

C) $\int_3^5 4\left(\frac{x^2}{2} - x + \frac{3}{2}\right) dx$

D) $\int_3^5 \left(\frac{x^2}{2} - x + \frac{3}{2}\right) dx$

Evaluate each definite integral.

10) $\int_{-3}^1 (-x^2 - 2x + 2) dx$

A) $\frac{20}{3} \approx 6.667$

B) $\frac{29}{3} \approx 9.667$

C) $\frac{5}{2} = 2.5$

D) 2

For each problem, find $F'(x)$.

12) $F(x) = \int_1^x (-t^2 + 4t - 1) dt$

A) $F'(x) = x^2 + 4x - 2$

B) $F'(x) = -x^2 + 4x - 1$

C) $F'(x) = x^2 + 4x$

D) $F'(x) = -x^2 + 8x - 10$

Express each definite integral in terms of u , but do not evaluate.

13) $\int_{-2}^0 -\frac{18x}{(3x^2 + 2)^2} dx; \ u = 3x^2 + 2$

A) $\int_{14}^2 -\frac{3}{u^2} du$

B) $\int_{10}^2 -\frac{3}{u^2} du$

C) $\int_5^2 -\frac{3}{u^2} du$

D) $\int_{14}^5 -\frac{3}{u^2} du$

Find the general solution of each differential equation.

14) $\frac{dy}{dx} = \frac{x^2}{e^{2y}}$

A) $\frac{e^{2y}}{2} = x^3 + C_1$
 $y = \frac{\ln(2x^3 + C)}{2}$

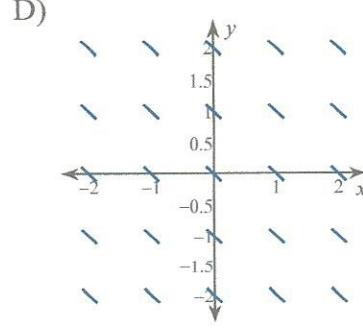
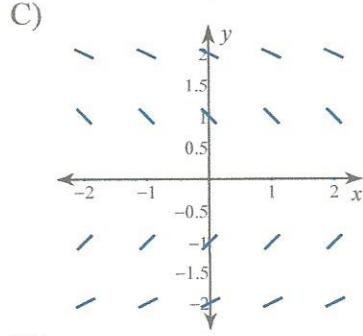
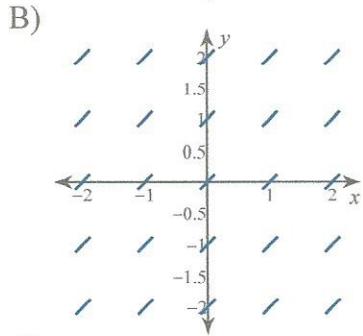
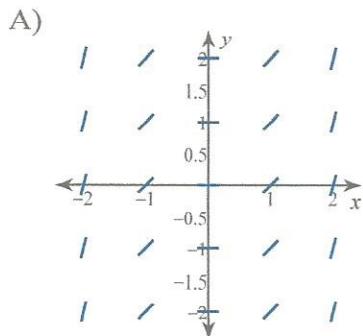
B) $\frac{e^{2y}}{2} = \frac{x^3}{3} + C$
 $y = \frac{\ln\left(\frac{2x^3}{3} + C\right)}{2}$

C) $\tan y = x + C$
 $y = \tan^{-1}(x + C)$

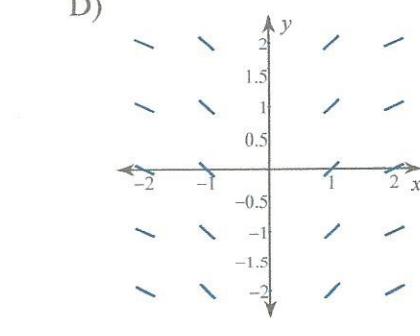
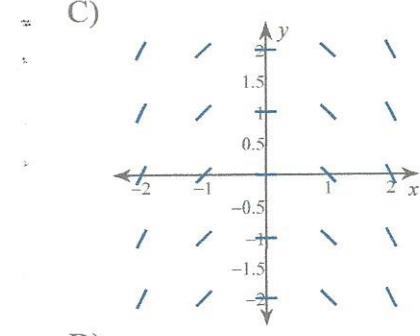
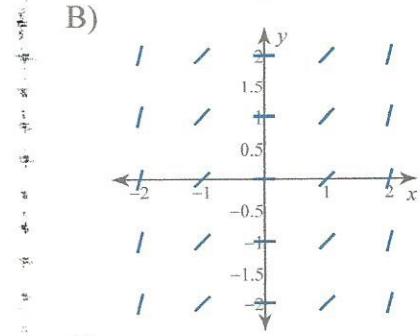
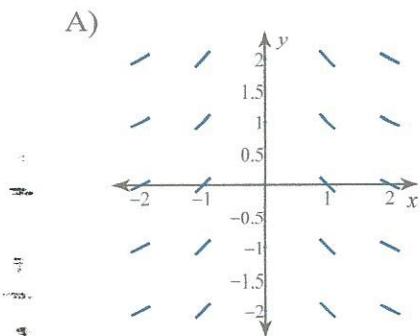
D) $\frac{y^3}{3} = \frac{x^3}{3} + C_1$
 $y = \sqrt[3]{x^3 + C}$

Sketch the slope field for each differential equation.

15) $\frac{dy}{dx} = -1$



16) $\frac{dy}{dx} = -\frac{1}{x}$



Evaluate each definite integral.

17) $\int_{-3}^0 (-2x - 2) dx$

- A) 8 B) 3
C) -7 D) 1

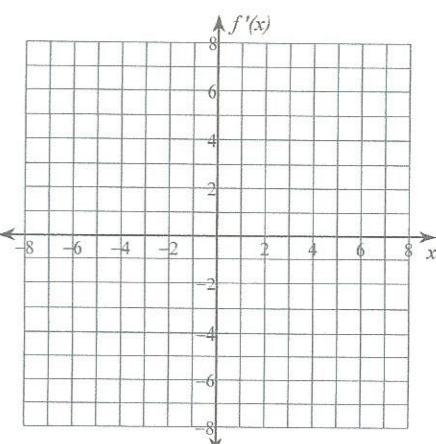
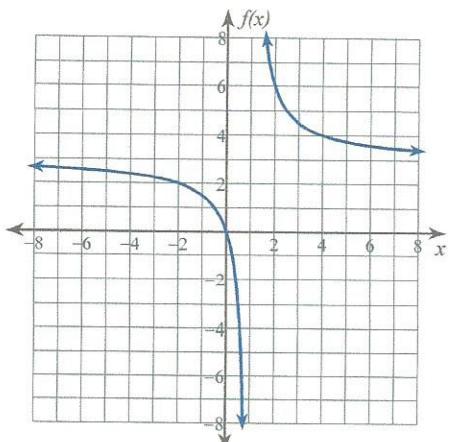
For each problem, find $F'(x)$.

18) $F(x) = \int_0^x (t^2 - 4t + 2) dt$

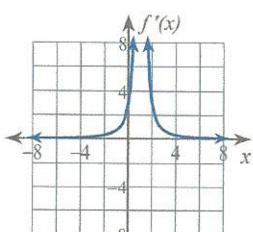
- A) $F'(x) = x^2 - 4$
B) $F'(x) = -x^2 - 6x - 11$
C) $F'(x) = x^2 - 2x + 1$
D) $F'(x) = x^2 - 4x + 2$

Given the graph of $f(x)$, sketch an approximate graph of $f'(x)$.

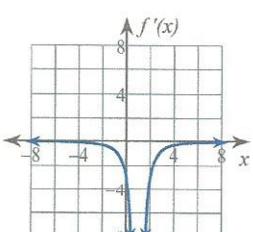
19)



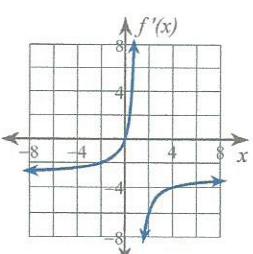
A)



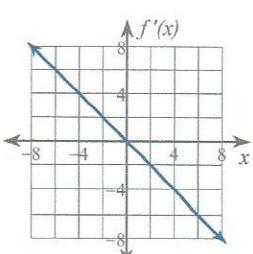
B)



C)



D)



Evaluate each indefinite integral using integration by parts. u and dv are provided.

20) $\int x \ln x^2 dx; u = \ln x^2, dv = x dx$

- A) $\frac{e^x}{2x+2} + C$
- B) $\frac{x^2 \ln x^2 - x^2}{2} + C$
- C) $\frac{(\ln 2x)^3}{3} + C$
- D) $\frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$

21) $\int x \sin x dx; u = x, dv = \sin x dx$

- A) $x \tan x + \ln |\cos x| + C$
- B) $\frac{x^2 \tan^{-1} x - x + \tan^{-1} x}{2} + C$
- C) $x \cos^{-1} x - (1 - x^2)^{\frac{1}{2}} + C$
- D) $-x \cos x + \sin x + C$

Evaluate each limit. Use L'Hôpital's Rule if it can be applied. If it cannot be applied, evaluate using another method and write a * next to your answer.

22) $\lim_{x \rightarrow 0^+} \frac{2(e^x + e^{-x})}{x}$

- A) ∞ * B) 0
C) -1 D) 2

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity function $v(t)$ and the acceleration function $a(t)$.

23) $s(t) = t^3 - 18t^2 + 81t$

- A) $v(t) = -3t^2 + 24t, a(t) = -6t + 24$
B) $v(t) = -3t^2 + 28t, a(t) = -6t + 28$
C) $v(t) = -3t^2 + 48t - 144, a(t) = -6t + 48$
D) $v(t) = 3t^2 - 36t + 81, a(t) = 6t - 36$

For each problem, find the average value of the function over the given interval.

24) $f(x) = 2x + 2; [-3, 0]$

- A) 11 B) 5
C) -1 D) 3

Solve each related rate problem.

25) A hypothetical square grows so that the length of its sides are increasing at a rate of 7 m/min. How fast is the area of the square increasing when the sides are 6 m each?

A) A = area of square s = length of sides t = time

$$\text{Equation: } A = s^2 \quad \text{Given rate: } \frac{ds}{dt} = 7 \quad \text{Find: } \frac{dA}{dt} \Bigg|_{s=6}$$

$$\frac{dA}{dt} \Bigg|_{s=6} = 2s \cdot \frac{ds}{dt} = 83 \text{ m}^2/\text{min}$$

B) A = area of square s = length of sides t = time

$$\text{Equation: } A = s^2 \quad \text{Given rate: } \frac{ds}{dt} = 7 \quad \text{Find: } \frac{dA}{dt} \Bigg|_{s=6}$$

$$\frac{dA}{dt} \Bigg|_{s=6} = 2s \cdot \frac{ds}{dt} = 77 \text{ m}^2/\text{min}$$

C) A = area of square s = length of sides t = time

$$\text{Equation: } A = s^2 \quad \text{Given rate: } \frac{ds}{dt} = 7 \quad \text{Find: } \frac{dA}{dt} \Bigg|_{s=6}$$

$$\frac{dA}{dt} \Bigg|_{s=6} = 2s \cdot \frac{ds}{dt} = 92 \text{ m}^2/\text{min}$$

D) A = area of square s = length of sides t = time

$$\text{Equation: } A = s^2 \quad \text{Given rate: } \frac{ds}{dt} = 7 \quad \text{Find: } \frac{dA}{dt} \Bigg|_{s=6}$$

$$\frac{dA}{dt} \Bigg|_{s=6} = 2s \cdot \frac{ds}{dt} = 84 \text{ m}^2/\text{min}$$

For each problem, find all points of relative minima and maxima.

26) $y = -\frac{x^2}{2} - x + \frac{1}{2}$

- A) No relative minima.

Relative maximum: $(-1, 1)$

- B) No relative minima.

No relative maxima.

- C) Relative minimum: $\left(-4, -\frac{7}{2}\right)$

No relative maxima.

- D) No relative minima.

Relative maximum: $\left(-\frac{1}{3}, \frac{7}{9}\right)$

Answers to Final Exam Review

- | | | | |
|-------|-------|-------|-------|
| 1) B | 2) B | 3) A | 4) D |
| 5) B | 6) A | 7) A | 8) D |
| 9) D | 10) A | 11) D | 12) B |
| 13) A | 14) B | 15) D | 16) A |
| 17) B | 18) D | 19) B | 20) B |
| 21) D | 22) A | 23) D | 24) C |
| 25) D | 26) A | | |