Calculus BC - 2022 AP Live Review Session 4
Convergent and Divergent Series

Type of Series	Form of Series	Convergence/Divergence Info	Conditions/Comments
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	If $0 < \lvert r \rvert < 1$, then the series converges. If $\lvert r \rvert \geq 1$, then the series diverges.	This sum of this type of infinite series is $\frac{a}{1-r}$. Be careful when finding the sum for a series starting at a value other than 0.
*n*th Term Test for Divergence	$\sum_{n=1}^{\infty} a_n$	If $\lim_{n\to\infty} a_n \neq 0$, then the series diverges.	If $\lim_{n\to\infty} a_n = 0$, then the *n*th term test is inconclusive and another test must be used.
Integral Test	$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} f(n)$	If $\int_1^{\infty} f(x)\,dx$ is a positive, finite value, then the series converges. If $\int_1^{\infty} f(x)\,dx$ diverges, then the series diverges.	$f(x)$ must be positive, continuous and decreasing
p-series Test	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	If $p > 1$, then the series converges. If $p \leq 1$, then the series diverges.	A series in this form with $p = 1$ is called the harmonic series.
Direct Comparison Test	$\sum_{n=1}^{\infty} a_n$	If $0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ is known to converge, then $\sum_{n=1}^{\infty} a_n$ converges. If $0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ is known to diverge, then $\sum_{n=1}^{\infty} a_n$ diverges.	The series must all have positive terms.
Limit Comparison Test	$\sum_{n=1}^{\infty} a_n$	If $a_n > 0$, $b_n > 0$, $\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ is known to converge, then $\sum_{n=1}^{\infty} a_n$ converges.	The series must all have positive terms. The reciprocal will have the same result.

Limit Comparison Test (cont'd)	$\sum_{n=1}^{\infty} a_n$	If $a_n > 0,\ b_n > 0,\ \lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ is known to diverge, then $\sum_{n=1}^{\infty} a_n$ diverges.	The series must all have positive terms. The reciprocal will have the same result.
Alternating Series Test	$\sum_{n=1}^{\infty} (-1)^n a_n$ or $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$	If $\lim_{n\to\infty} a_n = 0$ and $\lvert a_{n+1} \rvert < \lvert a_n \rvert$, then the series converges.	The Error Bound is $\lvert S - S_n \rvert < \lvert a_{n+1} \rvert$. The error of estimating the infinite sum by using the first *n* terms is less than the first omitted term.
Ratio Test	$\sum_{n=1}^{\infty} a_n$	If $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert < 1$, then $\sum_{n=1}^{\infty} a_n$ converges. If $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert > 1$, then $\sum_{n=1}^{\infty} a_n$ diverges.	If $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = 1$, then the ratio test is inconclusive and another test must be used.

Topic Name	Topic #
Geometric Series	10.2
*n*th Term Test for Divergence	10.3
Integral Test	10.4
p-Series	10.5
Direct Comparison/Limit Comparison Test	10.6
Alternating Series Test	10.7
Ratio Test	10.8

Multiple Choice Practice

1. Level: AP2

Given the series $\sum_{n=1}^{\infty} \frac{a_n 5^n}{n!}$, which of the following is the could be the correct set-up and conclusion for the ratio test?

(A) $\lim_{n\to\infty} \left| \frac{a_n 5^n}{n!} \cdot \frac{(n+1)!}{a_{n+1} 5^{n+1}} \right| > 1$; the series converges.

(B) $\lim_{n\to\infty} \left| \frac{a_{n+1} 5^{n+1}}{(n+1)!} \cdot \frac{n!}{a_n 5^n} \right| < 1$; the series diverges.

(C) $\lim_{n\to\infty} \left| \frac{a_{n+1} 5^{n+1}}{(n+1)!} \cdot \frac{n!}{a_n 5^n} \right| > 1$; the series converges.

(D) $\lim_{n\to\infty} \left| \frac{a_{n+1} 5^{n+1}}{(n+1)!} \cdot \frac{n!}{a_n 5^n} \right| < 1$; the series diverges.

2. Level: AP2

Let $\sum_{n=1}^{\infty} a_n$ be an infinite series with $a_n = f(n)$. Which of the following conditions must be met to apply the integral test?

(A) $f(x)$ is differentiable

(B) $f(x)$ is positive

(C) a_n is continuous

(D) a_n is decreasing

3. Level: AP2

Given the series $\sum_{n=1}^{\infty} (-1)^n a_n$, which of the following conditions must be met in order to use the Alternating Series Test?

I. $a_{n+1} \le a_n$ II. $a_n > 0$ III. $\lim_{n\to\infty} a_n = 0$

(A) I only
(B) I and III only
(C) III only
(D) I, II, and III

4. **Level: AP2**

Given the series $\sum_{n=1}^{\infty}\frac{n^2+3n-2}{n^3-n^2+1}$, which of the following series can be used as a comparison for the Limit Comparison test.

(A) $\sum_{n=1}^{\infty}\frac{1}{n^3}$

(B) $\sum_{n=1}^{\infty}\frac{1}{n^2}$

(C) $\sum_{n=1}^{\infty}n$

(D) $\sum_{n=1}^{\infty}\frac{1}{n}$

5. **Level: AP2**

Which of the following infinite series diverges by the n-th term test?

(A) $\sum_{n=1}^{\infty}\ln(n)$

(B) $\sum_{n=1}^{\infty}\frac{\ln(n)}{n}$

(C) $\sum_{n=1}^{\infty}\frac{n^3}{2^n}$

(D) $\sum_{n=1}^{\infty}\frac{3n^2+n-4}{n^3+2n}$

6. **Level: AP3**

Which of the following converge?

(A) $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$

(B) $\sum_{n=1}^{\infty}\frac{-1}{n}$

(C) $\sum_{n=1}^{\infty}\frac{n^{-2}}{\sqrt[3]{n}}$

(D) $\sum_{n=1}^{\infty}\frac{\sqrt[3]{n^4}}{6n^2}$

7. Level: AP3

$\sum_{n=1}^{\infty} 4\left(\frac{2}{3}\right)^{n-1}$ is

(A) 6

(B) 8

(C) 12

(D) divergent

8. Level: AP3

Which of the following statements about the series $\sum_{n=3}^{\infty} \frac{n^2-2}{n^3+4n^2+1}$ is true?

(A) The series converges by the limit comparison test with $\sum_{n=3}^{\infty} \frac{1}{n}$.

(B) The series converges by the limit comparison test with $\sum_{n=3}^{\infty} \frac{1}{n^3}$.

(C) The series diverges by the limit comparison test with $\sum_{n=3}^{\infty} \frac{1}{n}$.

(D) The series diverges by the limit comparison test with $\sum_{n=3}^{\infty} \frac{1}{n^3}$.

9. Level: AP3

Which of the following statements describes the series $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+3}$ using the alternating series test?

(A) converges absolutely

(B) converges conditionally

(C) diverges

(D) alternating series test is inconclusive

10. Level: AP3

Which of the following series converge?

I. $\sum_{n=0}^{\infty} \frac{3}{2^n}$ II. $\sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{2}}}$ III. $\sum_{n=1}^{\infty} (-1)^n$

(A) I only

(B) II only

(C) I and II only

(D) I, II, and III

11. Level: AP3

Let $\sum_{n=1}^{\infty} a_n$ be a series whose n^{th} partial sum is $S_n = \frac{2n+1}{5n-3}$. Which of the following statements is true?

(A) The series $\sum_{n=1}^{\infty} a_n$ diverges by the nth term test.

(B) The series $\sum_{n=1}^{\infty} a_n$ diverges by the p-series test.

(C) The series $\sum_{n=1}^{\infty} a_n$ converges because $\lim_{n\to\infty} S_n \neq 0$.

(D) The series $\sum_{n=1}^{\infty} a_n$ converges to $\frac{2}{5}$.

12. Level: AP4

For which of the following values of x does the series $\sum_{n=1}^{\infty} \left(\frac{x-2}{3}\right)^{n-1}$ converge?

(A) $x = -1$

(B) $x = 1$

(C) $x = -5$

(D) $x = 5$

13. Level: AP4

When applied to the series $\sum_{n=1}^{\infty} a_n$, the ratio test yields the result $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$. Which of the following *could* be true?

(A) $L = \frac{1}{2}$ and $\sum_{n=1}^{\infty} a_n$ converges conditionally.

(B) $L = \frac{2}{3}$ and $\sum_{n=1}^{\infty} a_n$ diverges.

(C) $L = 1$ and $\sum_{n=1}^{\infty} a_n$ converges conditionally.

(D) $L = \frac{3}{2}$ and $\sum_{n=1}^{\infty} a_n$ converges absolutely.

14. Level: AP4

Which of the following series converge conditionally?

I. $\sum_{n=0}^{\infty} (-1)^n \left(\frac{2}{3}\right)^n$ II. $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{n}\right)^{3/2}$ III. $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{2}{n}\right)$

(A) I and II only
(B) I and III only
(C) III only
(D) I, II, and III

15. Level: AP3

Which of the following geometric series converge?

I. $\sum_{n=0}^{\infty} \left(\frac{\pi}{3}\right)^n$ II. $\sum_{n=0}^{\infty} (0.99)^n$ III. $\sum_{n=1}^{\infty} 2\left(\frac{\pi}{e+1}\right)^n$

(A) II only
(B) I and II only
(C) II and III only
(D) I, II, and III

16. Level: AP3

Which of the following series converges?

(A) $\sum_{n=1}^{\infty} \frac{(-1)^{2n-1}}{n}$

(B) $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n+1}{3n-2}$

(C) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt[3]{n}}$

(D) $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{2^n}{n^{99}}$

17. Level: AP3

Which of the following series converge?

I. $\sum_{k=4}^{\infty} \frac{k^2+k}{\sqrt{k-3}}$ II. $\sum_{k=1}^{\infty} \frac{k+3}{(k-2)^2+1}$ III. $\sum_{k=1}^{\infty} \frac{6k+3}{k^3+k}$

(A) I and III only

(B) I and II only

(C) III only

(D) II and III only

18. Level: AP4

Let $a_n = \sqrt{n}$. Which of the following series converges?

(A) $\sum_{n=1}^{\infty} \frac{(5na_n+3)^4}{2n^6+3n^5}$

(B) $\sum_{n=1}^{\infty} \frac{n!}{2^n \cdot a_n}$

(C) $\sum_{n=1}^{\infty} \frac{5n-1}{n \cdot a_n}$

(D) $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot a_n}{n}$

19. Level: AP4

Which of the following series can be shown to converge by the alternating series test?

I. $\sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n^2}$ II. $\sum_{n=0}^{\infty} \frac{\cos(\pi n)}{\sqrt{n}}$ III. $\sum_{n=1}^{\infty} (-1)^{n-1} \cdot \frac{\sin(n)}{n}$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

20. Level: AP4

Which of the following series can be shown to converge by the alternating series test?

I. $\frac{1}{2} - 2 + \frac{27}{8} - 4 + \frac{125}{32} + \ldots + a_n + \ldots$, where $a_n = (-1)^{n-1} \cdot \frac{n^3}{2^n}$.

II. $1 - \frac{1}{8} - \frac{1}{18} + \frac{1}{16} - \frac{1}{50} - \frac{1}{72} + \frac{1}{49} - \frac{1}{128} - \frac{1}{162} + \ldots + a_n + \ldots$, where $a_n = \frac{\cos\left(\frac{2\pi n}{3}\right)}{(n+1)^2}$.

III. $\frac{1}{2} - \frac{\sqrt[3]{2}}{5} + \frac{\sqrt[3]{3}}{8} - \frac{\sqrt[3]{4}}{11} + \frac{\sqrt[3]{5}}{14} - \frac{\sqrt[3]{6}}{17} + \ldots + a_n + \ldots$, where $a_n = (-1)^{n+1} \frac{\sqrt[3]{n}}{3n-1}$.

(A) II only

(B) III only

(C) I and III only

(D) II and III only

21. Level: AP

Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{3^n n^{100}}{n!}$ II. $\sum_{n=1}^{\infty} \frac{3n-1}{7n}$ III. $\sum_{n=1}^{\infty} \left(\frac{\pi-1}{e}\right)^n$

(A) I only (B) III only (C) I and III only (D) I, II, and III

22. Level: AP4

What is the sum of the series $\frac{3}{\pi} - \frac{3}{2\pi} + \frac{3}{4\pi} - \frac{3}{8\pi} + \ldots + \left(\frac{-1}{2}\right)^n \frac{3}{\pi} + \ldots$?

(A) $\frac{2}{\pi}$

(B) $\frac{6}{\pi}$

(C) $\frac{6}{2\pi+1}$

(D) The series diverges.

23. Level: AP4

Consider the series $\sum_{n=2}^{\infty} a_n$ where $f(n) = a_n$. Which of the following statements about $f(x)$ and $\sum_{n=2}^{\infty} a_n$ could be true?

(A) $f(x) = \frac{1}{x}$ and $\sum_{n=2}^{\infty} a_n$ converges by the integral test.

(B) $f(x) = \frac{1}{2^x}$ and $\sum_{n=2}^{\infty} a_n$ converges by the integral test.

(C) $f(x) = xe^x$ and $\sum_{n=2}^{\infty} a_n$ diverges by the integral test.

(D) $f(x) = \frac{1}{x(\ln(x))^2}$ and $\sum_{n=2}^{\infty} a_n$ diverges by the integral test.

24. Level: AP4

If $0 < a_n < b_n$ where a_n and b_n are both decreasing sequences for $n \geq 0$, which of the following must be true?

(A) If $\sum_{n=0}^{\infty} a_n$ converges, then $\sum_{n=0}^{\infty} b_n$ converges.

(B) If $\sum_{n=0}^{\infty} b_n$ diverges, then $\sum_{n=0}^{\infty} a_n$ converges.

(C) If $\lim_{n\to\infty} b_n = 0$, then $\sum_{n=0}^{\infty} (-1)^n a_n$ converges.

(D) If $\sum_{n=0}^{\infty} (-1)^n b_n$ converges, then $\sum_{n=0}^{\infty} a_n$ converges.

25. Level: AP4

Given $\sum_{n=1}^{\infty} (-1)^n \frac{n^p}{\sqrt{n^2 - 3}}$, for what value(s) of p will the series be conditionally convergent?

(A) $p \geq 1$

(B) $p = 1$

(C) $\frac{1}{2} < p < \frac{3}{2}$

(D) $0 < p < 1$

26. Level: AP4

Consider the series $\sum_{n=1}^{\infty} a_n$ where $\lim_{n\to\infty} a_n = \frac{1}{2}$. Which of the following must be true?

(A) The series $\sum_{n=1}^{\infty} a_n$ diverges by the *n*th term test.

(B) The series $\sum_{n=1}^{\infty} a_n$ converges by the *n*th term test.

(C) The series $\sum_{n=1}^{\infty} a_n$ converges by the ratio test.

(D) The series $\sum_{n=1}^{\infty} a_n$ could converge or diverge.

27. Level: AP5

The sum of the geometric series $\sum_{n=1}^{\infty} 4\left(\frac{-2}{k}\right)^n$ is -1 where k is a constant. What is the value of k?

(A) $k = -\frac{2}{5}$ (B) $k = 6$ (C) $k = 8$ (D) $k = 10$

28. Level: AP5

Which of the following is true when the integral test is applied to the series $\sum_{n=1}^{\infty} ne^{-n}$ where $f(x) = xe^{-x}$?

(A) $\int_1^{\infty} f(x)\,dx = \frac{2}{e}$ and $\sum_{n=1}^{\infty} ne^{-n}$ converges.

(B) $\int_1^{\infty} f(x)\,dx$ converges and $\sum_{n=1}^{\infty} ne^{-n} = \int_1^{\infty} f(x)\,dx$.

(C) $\int_1^{\infty} f(x)\,dx$ diverges and the series $\sum_{n=1}^{\infty} ne^{-n}$ diverges.

(D) The conditions for the integral test are not met and the integral test cannot be applied.

29. Level: AP5

Which of the following could be a valid conclusion of the integral test for the series $\sum_{n=1}^{\infty} a_n$ where $f(n) = a_n$ and $f(n)$ is known to be positive and decreasing ?

(A) The series $\sum_{n=1}^{\infty} a_n$ diverges because $f(x)$ is not continuous over the interval $[1, \infty)$.

(B) The series $\sum_{n=1}^{\infty} a_n$ diverges because $\int_1^{\infty} f(x)dx \neq 0$.

(C) The series $\sum_{n=1}^{\infty} a_n$ converges to 2 because $f(x)$ is differentiable over the interval $[1, \infty)$ and $\int_1^{\infty} f(x)dx = 2$.

(D) The series $\sum_{n=1}^{\infty} a_n$ converges because $f(x)$ is continuous over the interval $[1, \infty)$ and $\int_1^{\infty} f(x)dx = 8$.

30. Level: AP5

For which value(s) of k do both $\sum_{n=1}^{\infty} \frac{n^3}{n^{k^2}}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{2}+1\right)^{-n}$ converge?

(A) $-\infty < k < -2$

(B) $-4 < k < -\sqrt{3}$

(C) $k < -4$ or k > 2

(D) $-4 < k < 0$ and $k > \sqrt{3}$