# Derivatives and Chain Rule

**1.** If $y = \sqrt{x}e^x$, then $\frac{dy}{dx} =$

(A) $\frac{1}{2\sqrt{x}}e^x$ (B) $\frac{1}{2\sqrt{x}}e^x + x\sqrt{x}e^{x-1}$ (C) $\frac{1}{2\sqrt{x}}e^x - \sqrt{x}e^x$ (D) $\frac{1}{2\sqrt{x}}e^x + \sqrt{x}e^x$

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**2.** $\frac{d}{dx}\left(\frac{x-\ln x}{x^2+1}\right)\Big|_{x=1} =$

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) $-\frac{3}{4}$

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**3.** If $x^3 - 2xy - y^2 = -4$, what is $\frac{dy}{dx}$ at the point $(-1, 3)$?

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) $-\frac{3}{4}$

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**4.** $\frac{d}{dx}\left(5\left(\cos\sqrt{x}\right)^2\right) =$

(A) $\frac{-5\cos\sqrt{x}\sin\sqrt{x}}{\sqrt{x}}$ (B) $-10\cos\sqrt{x}\sin\sqrt{x}$ (C) $-10\sin\left(\frac{1}{2\sqrt{x}}\right)$ (D) $-\frac{5\sin\sqrt{x}}{\sqrt{x}}$

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**5.** If $f(x) = e^4 + e^{4x} + x^4 + 4^x$, then $f'(x) =$

(A) $e^4 + 4e^{4x} + 4x^3 + 4^x$ (B) $4e^{4x} + 4x^3 + 4^x\ln 4$ (C) $e^{4x} + 4x^3 + 4^x\ln 4$ (D) $4e^{4x} + 4x^3 + 4^x$

**6.** If $P(t) = 2\sin t$, then find $P^{(19)}\left(\frac{2\pi}{3}\right)=$

(A) $-\sqrt{3}$ (B) $-1$ (C) $1$ (D) $\sqrt{3}$

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**7.** $\frac{d^2y}{dx^2} = 3x^2 + 7x^3 - 2x^4$; find $\left.\frac{d^5y}{dx^5}\right|_{x=2}$.

(A) $-54$ (B) $-48$ (C) $-42$ (D) $0$

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**8.** If $y = \arctan(2x)$, $\frac{dy}{dx} =$

(A) $2\text{arcsec}^2(2x)$ (B) $\frac{2}{\sqrt{1-4x^2}}$ (C) $\frac{1}{1+4x^2}$ (D) $\frac{2}{1+4x^2}$

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**9.**

| $x$ | $g(x)$ | $g'(x)$ |
|---|---|---|
| 2 | 3 | 4 |
| 3 | 5 | 1 |
| 4 | 6 | 3 |

The table gives selected values for a differentiable and increasing function $g$ and its derivative. If $g^{-1}$ is the inverse function of $g$, what is the value of $(g^{-1})'(3)$?

(A) 1 (B) $\frac{1}{4}$ (C) $-1/4$ (D) $-1$

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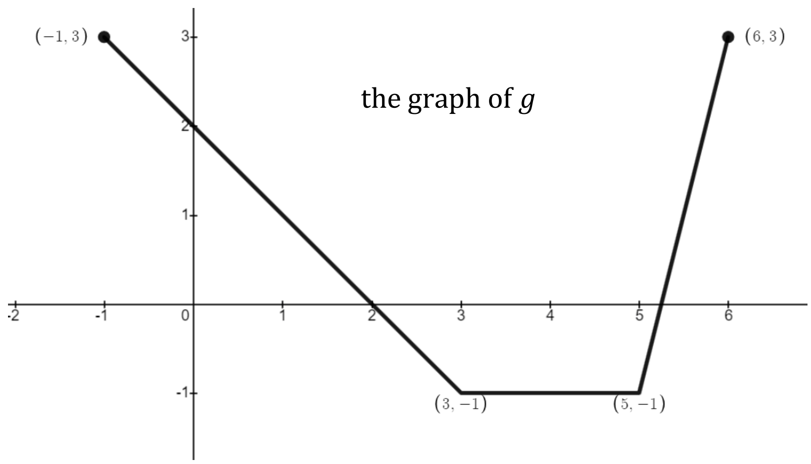
**10.** Find the slope of the tangent line of $h(x) = \frac{4}{x} - \sqrt[3]{2x}$ at $x = 4$.

(A) $-\frac{23}{6}$ (B) $-\frac{5}{12}$ (C) $-\frac{1}{3}$ (D) $4\ln 4 - \frac{1}{6}$

**11.**

| $x$ | $f(x)$ | $f'(x)$ |
|---|---|---|
| 1 | $\frac{1}{3}$ | $\pi$ |
| 2 | 7 | $-2$ |
| 4 | 5.5 | 3 |




Selected values of a continuous differentiable function $f$ and its derivative, $f'$, are given above. Also shown is the graph of the continuous function $g$ made up of three line segments.

(a) Let $p(x) = f(x) + 3g(x)$. Find $p'(2)$.

(b) Let $h(x) = \frac{e^x}{1+g(x)}$. Find $h'(0)$.

(c) Let $l(x) = g(f(2x))$. Find $l'(2)$.

(d) Let $k(x) = 3f(x)g(x)$. Find $k'(1)$

(e) Write the equation of the tangent line to the graph of $f(2x)$ at $x = 1$.

**12.** Functions $f, g$, and $h$ are twice-differentiable functions with $g(1) = h(1) = 5$.

The line $y = 5 + \frac{3}{4}(x - 1)$ is tangent to both the graph of $g$ at $x = 1$ and the graph of $h$ at $x = 1$.

(a) Find $g'(1)$.

(b) Let $b$ be the function given by $b(x) = 2x^2 g(x)$. Write an expression for $b'(x)$. Find $b'(1)$.

(c) Let $w$ be the function given by $w(x) = \frac{4h(x) - x}{2x+1}$. Write an expression for $w'(x)$. Find $w'(1)$.

(d) Use the tangent line to the graph of $h(x)$ at $x = 1$ to approximate $h(0.8)$.

(e) Let $p$ be the function given by $p(x) = g(x) + f\left(\frac{x}{2}\right)$. Given that $p'(1) = \frac{5}{4}$, find $f'\left(\frac{1}{2}\right)$. Show your reasoning.