What Do We Need to Know?

Position Vector	$(x(t), y(t))$ or $\langle x(t), y(t)\rangle$	**Position at time *t***	$x(t) = x(a) + \int_a^t x'(t)\,dt$
Velocity Vector	$(x'(t), y'(t))$ or $\left(\frac{dx}{dt}, \frac{dy}{dt}\right)$	**Speed**	$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$
Acceleration Vector	$(x''(t), y''(t))$ or $\left(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}\right)$	**Total Distance Traveled from *t* = *a* to *t* = *b***	$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$
Slope of Tangent Line to a Curve	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$	**Second Derivative (Parametric)**	$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\frac{dx}{dt}}$

Finding Your Way Around AP Classroom

Topic Name	Topic #
Defining and Differentiating Parametric Equations	9.1
Second Derivatives of Parametric Equations	9.2
Finding Arc Lengths of Curves Given by Parametric Equatons	9.3
Defining and Differentiaitng Vector-Valued Functons	9.4
Interpreting Vector-Valued Functions	9.5
Solving Motion Problems Using Parametric and Vector-Vlaued Functions	9.6

Short Answer / Multiple Choice Practice

1. Level: AP3

The position of a particle at any time $t \geq 0$ is given by $\langle x(t),\, y(t) \rangle = \left\langle t^2 - 3,\, \frac{1}{2t-1} \right\rangle$. Find the acceleration vector at time $t = 2$.

(A) $\left\langle 2, \frac{8}{27} \right\rangle$ (B) $\left\langle 4, -\frac{2}{9} \right\rangle$ (C) $\left\langle 4, -\frac{1}{9} \right\rangle$ (D) $\left\langle 2, \frac{2}{27} \right\rangle$

2. Level: AP3 

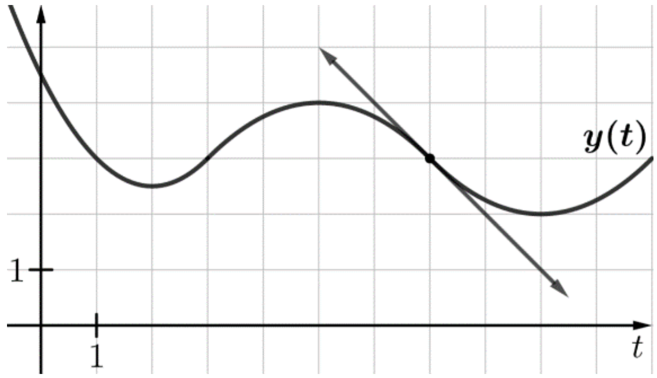
Given $\frac{dx}{dt} = \frac{e^{0.4t}}{\sin t}$ and $\frac{dy}{dt} = \frac{\cos^2\left(\frac{t}{5}\right)}{\sqrt{t}}$, find the slope of the tangent line to the curve $(x(t), y(t))$ at $t = 2$.

3. Level: AP4

For $t \geq 0$, a bug moves in the xy- plane with position $(x(t), y(t))$, where $x(t) = \sin\left(\frac{\pi t}{4}\right)$ and $y'(t) = t^2 - 6t + 8$. At which of the following times is the bug at rest?

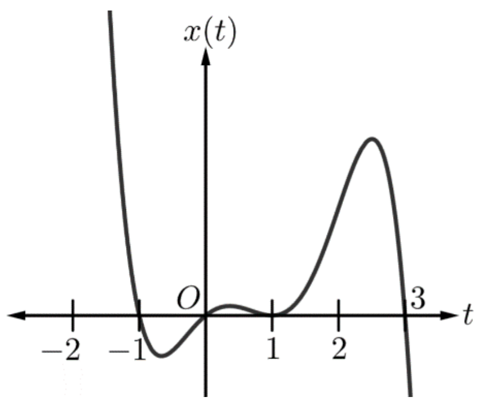
(A) $t = 2$ (B) $t = 3$ (C) $t = 4$ (D) $t = 8$

4. Level: AP4



A particle moves in the xy- plane with position $(x(t), y(t))$. The graph of $y(t)$ is shown in the figure above along with the line tangent to $y(t)$ at $t = 7$. If $\frac{dx}{dt} = (11 - 2t)^2$, find the speed of the particle at time $t = 7$.

5. Level: AP3



t	3
$y(t)$	0
$y'(t)$	2
$y''(t)$	−1

The position of a particle that moves in the xy- plane is given by the parametric functions $x(t)$ and $y(t)$. The graph of $x(t)$ is shown in the figure above along with values for $y(t), y'(t)$, and $y''(t)$ at $t = 3$. Which of the following describes the direction of motion for the particle at time $t = 3$?

(A) Directly up

(B) Up and to the right

(C) Up and to the left

(D) Down and to the left

6. Level: AP5

If $x(t) = 3t^2$ and $y(t) = 2t^4 + 3$, for $t > 0$, then $\dfrac{d^2y}{dx^2} =$

(A) $\dfrac{8}{3}t$ (B) $\dfrac{4}{9}t$ (C) $\dfrac{4}{9}$ (D) $\dfrac{3}{2t^3}$

7. Level: AP4 

The motion of a particle in the *xy*-plane at any time $t \geq 0$ can be described by $\dfrac{dx}{dt} = 1 - 2t$ and $y(t) = \dfrac{4}{3}t^{3/2}$. What is the total distance traveled by the particle from $t = 1$ to $t = 4$?

(A) 11.136

(B) 15.340

(C) 20.440

(D) 21.637

8. Level: AP5

For $t \geq 0$, the position of a particle moving in the xy- plane can be modeled by $\left(x(t), y(t)\right)$, where $y(t) = \dfrac{12 - 3t^2}{t^2 + 4t - 6}$ and $\dfrac{dx}{dt} = 4te^{-2t}$. At $t = 0$, the particle is at $(1, -2)$. As t increases without bound, what position will the particle approach?

(A) $(0, -3)$

(B) $(2, -3)$

(C) $(1, -5)$

(D) $(4, -1)$

9. Level: AP3

An object moving along a curve in the *xy*-plane has velocity vector $\left\langle \cos\left(t^2\right), 3-e^{t/3} \right\rangle$. Which of the following represents the acceleration of the object at time $t = 4$?

(A) $\left\langle 8\sin(16), -\frac{4}{3}e^{4/3} \right\rangle$

(B) $\left\langle -\sin(16), -e^{4/3} \right\rangle$

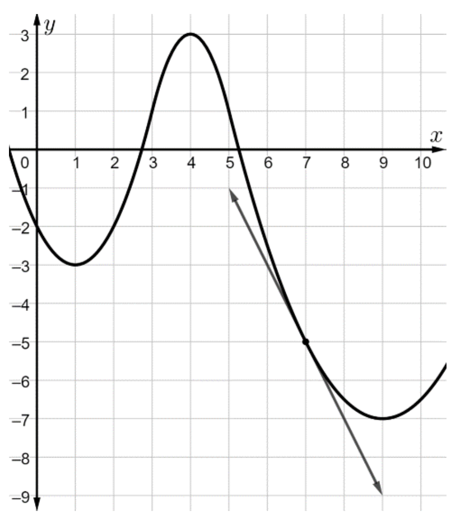
(C) $\left\langle -8\sin(16), -\frac{1}{3}e^{4/3} \right\rangle$

(D) $\left\langle -4\sin(16), -\frac{1}{3}e^{4/3} \right\rangle$

10. Level: AP4

The motion of a particle in the *xy*-plane at any time $t \geq 0$ can be described by a curve with the properties $x(t) = 4t^3 - 48t$ and $\frac{dy}{dt} = 10t - 2$. At which time is the line tangent to the path of the particle vertical?

(A) $t = 0$ (B) $t = \frac{1}{5}$ (C) $t = 2$ (D) $t = \sqrt{12}$



11. Level: AP4

The position of a particle that moves in the xy- plane is given by the parametric functions $x(t)$ and $y(t)$, where $y(t) = e^{4t-t^2}$. The path of the particle xy- plane is shown in the figure above. What is the value of $x'(7)$?

(A) $\frac{-1}{2e^{21}}$ (B) $\frac{2}{e^{21}}$ (C) $\frac{5}{e^{21}}$ (D) $\frac{20}{e^{21}}$

Free Response Practice

1. 

For $t \geq 0$, the position of a particle moving in the *xy*-plane is given by the vector $p(t) = \langle x(t), y(t) \rangle$ with $p(0) = (2, 4)$. It is known that $\dfrac{dx}{dt} = \dfrac{t^2 + \sin\left(t^{1.2} + 1\right)}{e^t}$ and $\dfrac{dy}{dt} = 2y\left(1 - \dfrac{y}{12}\right)$.

(a) Write the equation of the line tangent to the curve at time $t = 0$.

(b) Describe the direction of motion of the particle at time $t = 0$.

(c) Find the *x*-coordinate of the particle at time $t = 3$.

(d) Find $\lim\limits_{t \to \infty} y(t)$.

2.

t	0	1.5	2	4
$y''(t)$	2	–1	6	5

The position of a particle moving in the xy- plane is defined by the parametric equations $x(t)$ and $y(t)$, where $x(t) = y'(\sqrt{t})$. It is known that at $t = 0$, $x = 3$. The function $y(t)$ is not explicitly known, but selected values for $y''(t)$ are shown in the table above.

(a) Let S_R be the right Riemann sum approximation of $\int_0^4 y''(t)\,dt$ using the three intervals indicated in the table. Show that $S_R = \frac{23}{2}$. Use S_R to approximate $y'(4)$.

(b) For $t > 8$, the nth - derivative of $x(t)$ is defined as $x^{(n)}(t) = \frac{2^n}{n!} x^{(n-1)}(t)$, where $n \geq 1$. Using the approximation of $y'(4)$ from part (a), find the third-degree Taylor polynomial for $x(t)$ about $t = 16$.

(c) The slope of the line tangent to the path of the particle is -7 at $t = 16$. Use the Taylor polynomial for $x(t)$ about $t = 16$ found in part (b) to estimate $y'(16)$.

2.

t	0	1.5	2	4
$y''(t)$	2	–1	6	5

The position of a particle moving in the xy- plane is defined by the parametric equations $x(t)$ and $y(t)$, where $x(t) = y'\left(\sqrt{t}\right)$. It is known that at $t = 0$, $x = 3$. The function $y(t)$ is not explicitly known, but selected values for $y''(t)$ are shown in the table above.

(d) Use Euler's method with the data in the table, starting at $t = 0$ with two steps of equal size, to approximate $y'(3)$.

(e) For $0 \le t < 5$, the y - coordinate of the particle's position at time t can be modeled by $y(t) = \frac{1}{4}t - \sum_{n=1}^{\infty} \frac{(t-2)^n}{n \cdot 3^n}$. Find $y'(1)$. According to this model, does $y(t)$ have a relative minimum, relative maximum, or neither at $t = 1$? Give a reason for your answer.