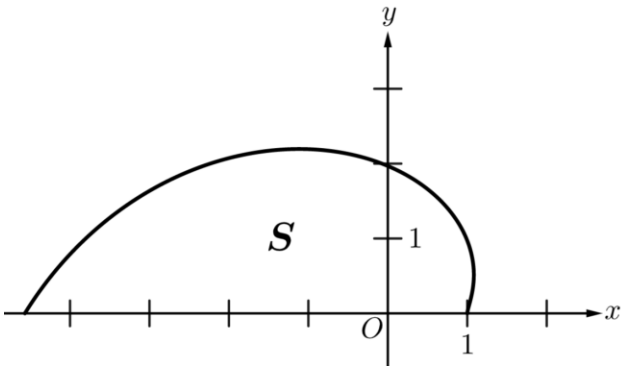
What Do We Need to Know?

Conversion Formulas	$x = r\cos(\theta) \quad y = r\sin(\theta)$	**Slope of the Tangent to a Polar Curve**	$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$
Area Bounded by a Polar Curve	$A = \frac{1}{2}\int_{\alpha}^{\beta}\left[f(\theta)\right]^2 d\theta$	**Distance to the Origin**	$r = f(\theta)$

Finding Your Way Around AP Classroom

Topic Name	Topic #
Defining Polar Coordinates and Differentiating in Polar Form	9.7
Find the Area of a Polar Region of the Area Bounded by a Single Polar Curve	9.8
Find the Area of the Region Bounded by Two Polar Curves	9.9

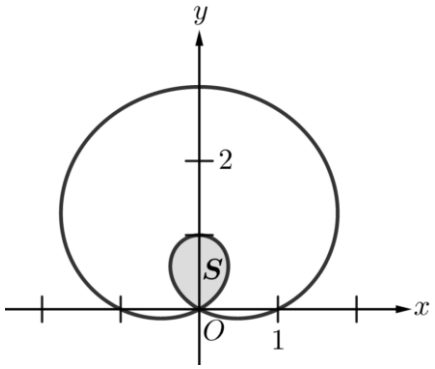
Short Response and Multiple-Choice Practice



1. Level: AP3

The figure above shows the graph of the polar curve $r = \theta^{1.5} + \cos(\theta)$ on the interval $0 \le \theta \le \pi$. Let S be the region bounded by r and the x-axis. Find the area of region S.

2. Level: AP4



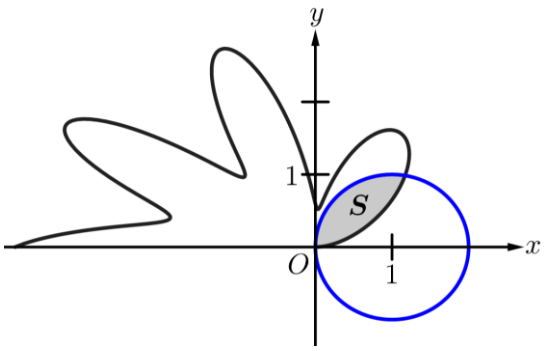
The graph of the polar curve $r = 1 + 2\sin(\theta)$ is shown in the figure above for $0 \le \theta \le 2\pi$. Let S be the shaded region in the inner loop of the polar curve as indicated above. Which of the following gives an expression for the area of region S?

(A) $\frac{1}{2}\int_0^{7\pi/6}\left(1+2\sin(\theta)\right)^2 d\theta$

(B) $\frac{1}{2}\int_{-\pi/6}^{7\pi/6}\left(1+2\sin(\theta)\right)^2 d\theta$

(C) $\frac{1}{2}\int_{7\pi/6}^{11\pi/6}\left(1+2\sin(\theta)\right)^2 d\theta$

(D) $\frac{1}{2}\int_0^{2\pi}\left(1+2\sin(\theta)\right)^2 d\theta$



3. Level: AP4 

The graphs of the polar curves $r = \theta + \sin\left(2\theta^2\right)$ and $r = 2\cos(\theta)$ are shown in the figure above for $0 \le \theta \le \pi$. Find the area of the shaded region S.

(A) 0.585 (B) 0.602 (C) 1.205 (D) 1.347

4. Level: AP4 

Let r be the polar curved defined by $r=\theta^2+\sin(3\theta)$. Find the values of both $r(\theta)$ and the y - coordinate of the polar curve when $\theta=\frac{4\pi}{3}$. Explain what these values represent in terms of the graph of r.

5. Level: AP4

Let r be the polar curved defined by $r=\theta-2\cos(\theta)$. On what open intervals, $0\le\theta\le 2\pi$, is r decreasing?

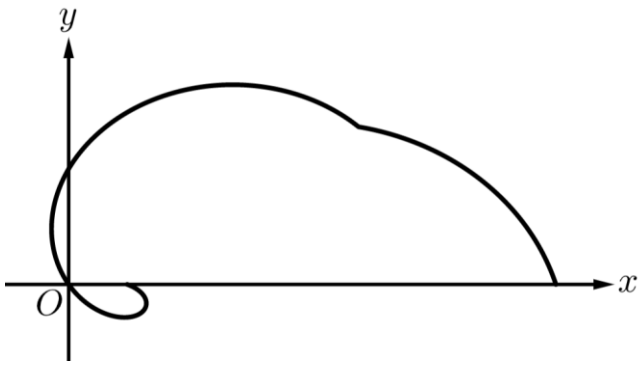
(A) $\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$ (B) $\left(\frac{7\pi}{6},\frac{11\pi}{6}\right)$ (C) $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ (D) $\left(0,\frac{7\pi}{6}\right)$ and $\left(\frac{11\pi}{6},2\pi\right)$

6. Level: AP4 

For $0\le\theta\le\pi$, a particle travels along the polar curve $r(\theta)=2\theta-4\cos(3\theta)$. Find the maximum distance from the origin of the particle?

7. Level: AP5

Let r be the polar curved defined by $r = \frac{3}{2}\theta + 2\cos\left(\frac{\theta^2}{2}\right)$, where $0 \le \theta \le \pi$. Find the value of θ when the x - coordinate is -3.



8. Level: AP5

The graph of the polar curve $r = 3\cos(\theta) + \left|\theta - \frac{\pi}{2} + 1\right|$ is shown in the figure above for $0 \le \theta \le \pi$. Which of the following statements are true about the graph of r at $\theta = \frac{5\pi}{6}$?

I. The graph of r is getting closer to the origin
II. The graph of r is getting closer to the x - axis
III. The graph of r is getting closer to the y - axis

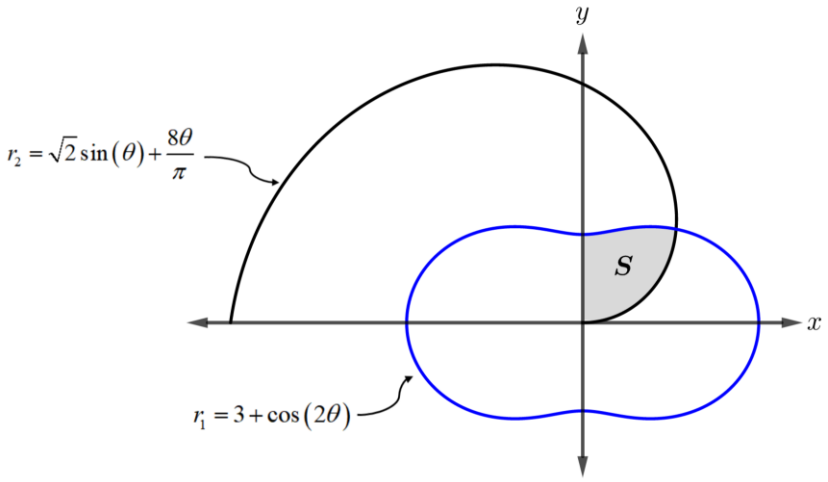
(A) I only (B) III only (C) I and II only (D) I, II, and III

9. Level: AP5

Find the slope of the line tangent to the graph of $f(\theta) = 1 + \cos\theta$ at $\theta = \frac{\pi}{6}$.

(A) $-\frac{1}{2}$ (B) $-\frac{\sqrt{3}+1}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) -1

Free Response Practice



1. The figure above shows the polar curves $r_1 = 3 + \cos(2\theta)$ for $0 \le \theta \le 2\pi$ and $r_2 = \sqrt{2}\sin(\theta) + \frac{8\theta}{\pi}$ for $0 \le \theta \le \pi$. Let S be the shaded region in the first quadrant bounded by graphs of r_1 and r_2 as shown in the figure above.

(a) Find the area of region S.

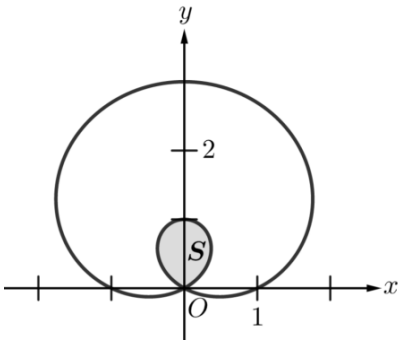
(b) Find $r_1'(\theta)$ when $\theta = \frac{4\pi}{11}$. What does this value indicate about the graph of r_1?

(c) For $0 \le \theta \le \pi$, find the value of θ for which the y-coordinate for r_2 is a maximum. Justify your answer.

(d) Let $(x(t), y(t))$ represent the position of a particle moving along the curve r_1 at time t, where $t = \theta$. Find the position of the particle at time $t = 2$.

Extra Practice

10. Level: AP5



The graph of the polar curve $r = 1 + 2\sin(\theta)$ is shown in the figure above for $0 \le \theta \le 2\pi$. Let S be the shaded region in the inner loop of the polar curve as indicated above. Which of the following gives an expression for the area of region S?

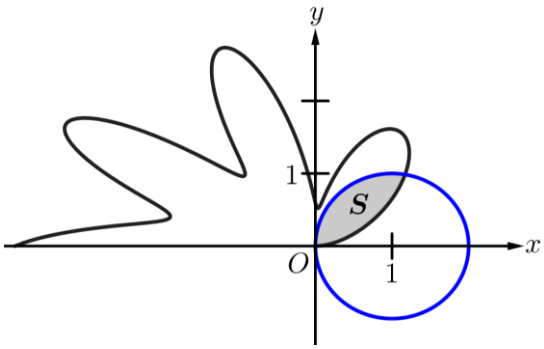
(A) $\frac{1}{2}\int_0^{7\pi/6}\left(1+2\sin(\theta)\right)^2 d\theta$

(B) $\frac{1}{2}\int_{-\pi/6}^{7\pi/6}\left(1+2\sin(\theta)\right)^2 d\theta$

(C) $\int_{7\pi/6}^{\pi/2}\left(1+2\sin(\theta)\right)^2 d\theta$

(D) $\int_{7\pi/6}^{3\pi/2}\left(1+2\sin(\theta)\right)^2 d\theta$

11. Level: AP4



The graphs of the polar curves $r = \theta + \sin\left(2\theta^2\right)$ and $r = 2\cos(\theta)$ are shown in the figure above for $0 \le \theta \le \pi$. At which value(s) of θ do the two curves intersect?

(A) $\theta = 0.6998$ only

(B) $\theta = 1.5299$ only

(C) $\theta = 0$ and 0.6998

(D) $\theta = 0$ and 1.5299

12. Level: AP3

Given the polar curve $r = 8\cos\theta$, which of the following could be θ if $x = 2$?

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$