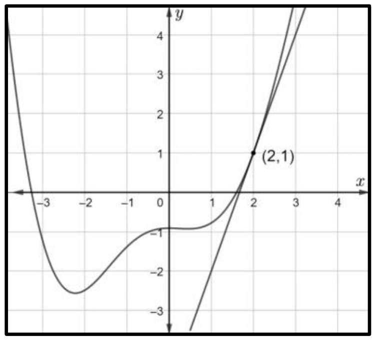
Calculus BC - 2022 AP Live Review Session 3
Taylor Polynomials
What Do We Need to Know ?

Taylor and Maclaurin Polynomials
General Equation for a Taylor Polynomial centered at $x = a$. $$T_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \frac{f'''(a)(x-a)^3}{3!} + \cdots + \frac{f^{(n)}(a)(x-a)^n}{n!}$$
General Equation for a Maclaurin Polynomial (centered at $x = 0$). $$P_n(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \cdots + \frac{f^{(n)}(0)x^n}{n!}$$

Open Response Practice

1. Level: AP3

Find the third-degree Taylor polynomial to the equation $f(x) = \ln x$ centered at $x = 1$.



2. Level AP4

A function f has derivatives of all orders for all values of x. The graph of f and the line tangent to the graph of f at $x = 2$ are shown in the figure above. Let g be the function defined by $g(x) = 3 + \int_2^x f(t)\,dt$.

Find the second-degree Taylor polynomial for $g(x)$ centered at $x = 2$.

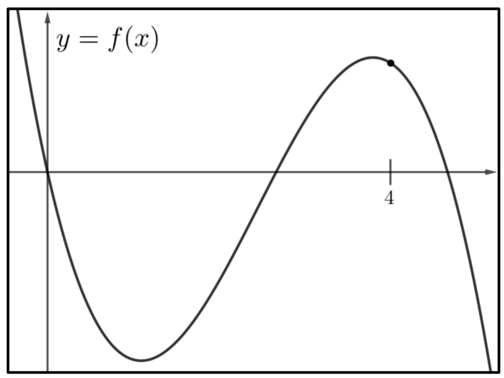
x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
1	-4	4	6	-8

3. Level: AP3

Let $P_3(x)$ be the third-degree Taylor polynomial for f centered at $x = 1$, where f has derivatives for all orders. Selected values of f and its derivatives at $x = 1$ are shown in the table above. What is the value of $P_3(0)$?

4. Level: AP3

Let $P_4(x) = 2 - 3(x+1) + 5(x+1)^2 - \frac{2}{3}(x+1)^3 + \frac{6}{5}(x+1)^4$ be the 4th degree Taylor polynomial for the function f about $x = -1$. What is the value of $f^{(4)}(-1)$?



5. Level: AP3

The figure above shows a portion of the graph of $f(x)$. Let $P_2(x)$ be the second-degree Taylor polynomial to f at $x = 4$. Explain why $P_2(x) \neq 5 - 2(x-4) + \frac{3(x-4)^2}{2}$.

Multiple Choice Practice

6. Level: AP2

Which of the following is a third-degree Taylor polynomial about $x = -3$?

(A) $T_3(x) = 4 - x + \frac{4x^2}{2!} - \frac{2x^3}{3!}$

(B) $T_3(x) = 4 - (x-3) + \frac{4(x-3)^2}{2!} - \frac{2(x-3)^3}{3!}$

(C) $T_3(x) = 4 - (x+3) + \frac{4(x+3)^2}{5!} - 2(x+3)^3$

(D) $T_3(x) = 4 - (x+3) + \frac{4(x+3)^2}{2!} - \frac{2(x+3)^3}{3!} + \ldots$

7. Level: AP3

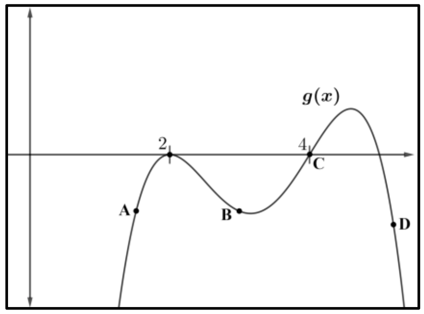
Let $P_4(x) = -4 + 5x - \frac{3}{2}x^2 + \frac{9}{4!}x^4$ be the fourth-degree Maclaurin polynomial for $g(x)$. Which of the following statements about $g(x)$ at $x = 0$ is true?

(A) $g(x)$ is increasing and concave up

(B) $g(x)$ is increasing and concave down

(C) $g(x)$ is decreasing and concave up

(D) $g(x)$ is decreasing and concave down



8. Level: AP4

Let $D_2(x) = -2 - 3(x-a) + 6(x-a)^2$ be the second-degree Taylor polynomial for g about $x = a$, where a is a constant. If the graph of g is shown in the figure above, then $x = a$ could be the x- coordinate for which of the following points?

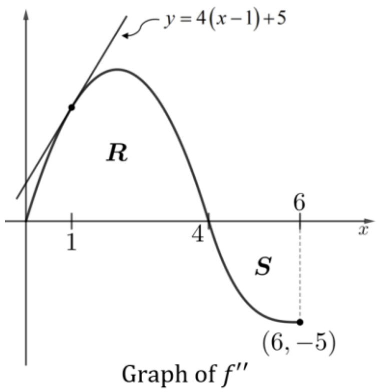
(A) Point *A*

(B) Point *B*

(C) Point *C*

(D) Point *D*

Free Response Practice



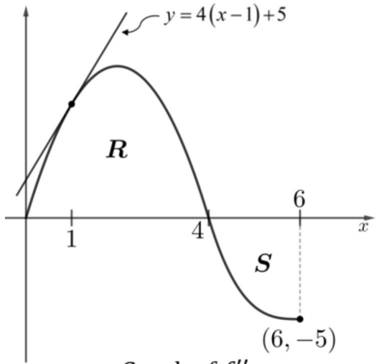
x	-2	-1	1
$f(x)$	7	-3	2
$f'(x)$	-2	$\frac{1}{2}$	$-\frac{1}{3}$

1. The function f has derivatives of all orders. Portions of the graph of f'' and the line tangent to f'' at $x=1$ are shown in the figure above. The graph of f'' has horizontal tangent lines at $x=2$ and $x=6$. The areas of the regions bounded by the x-axis and the graph of f'' on the intervals $[0,4]$ and $[4,6]$ are 14 and 5 respectively. Selected values for f and f' are given in the table above.

(a) Let $T_2(x)$ denote the second-degree Taylor polynomial for f about $x=-2$. It is known that $T_2(1)=4$. Find $f''(-2)$.

(b) Write the third-degree Taylor polynomial for f about $x=1$.

(c) Let g be a function whose derivative is defined by $g'(x)=\int_x^6 tf'''(t)\,dt$ and $P_3(x)=2+Ax+Bx^3$ is the third-degree Maclaurin polynomial for g, where A and B are constants. Find the values of A and B given that $f^{(3)}(0)=\frac{11}{2}$.



Graph of f''

x	-2	-1	1
$f(x)$	7	-3	2
$f'(x)$	-2	$\frac{1}{2}$	$-\frac{1}{3}$

1. The function f has derivatives of all orders. Portions of the graph of f'' and the line tangent to f'' at $x=1$ are shown in the figure above. The graph of f'' has horizontal tangent lines at $x=2$ and $x=6$. The areas of the regions bounded by the x-axis and the graph of f'' on the intervals $[0,4]$ and $[4,6]$ are 14 and 5 respectively. Selected values for f and f' are given in the table above.

(d) Let $h(x)=f\left(3-4x\right)$. Use Euler's method, with two steps of equal size starting at $x=1$, to approximate $h(0)$. Show the work that leads to your answer.

(e) Let $T_3(x)=51-4(x-11)+\frac{75}{2}(x-11)^2-\frac{1}{6}(x-11)^3$ be the third-degree Taylor polynomial for f centered at $x=11$. Justify why there must be a value k, for $1<k<11$, such that $f^{(3)}\left(x\right)=7$.

Multiple Choice Practice

9. Level: AP5

Let $P_3(x) = -2 + \frac{3x}{5} - \frac{9x^2}{50} + \frac{27x^3}{500}$ be the third-degree Maclaurin polynomial for $f(x)$ and let $\sum_{n=0}^{\infty} a_n$ be a geometric series where $\sum_{n=0}^{\infty} a_n = -\frac{5}{4}$. For which value of x does $P_3(x)$ give the first four terms for $\sum_{n=0}^{\infty} a_n$?

(A) $x = -2$

(B) $x = -1$

(C) $x = -\frac{3}{10}$

(D) $x = 2$

Open Response Practice

10. Level: AP5

Let $T_2(x)$ be the second-degree Taylor polynomial for g about $x = -1$ and let $y = 2x - 3$ be the equation of the line tangent to the graph of g at $x = -1$. If $T_2(3) = -4$, what is the value of $g''(-1)$?

11. Level: AP5

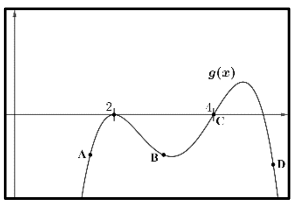
How many terms will it take to approximate $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$ with an error less than or equal to 0.05?

12. Level: AP5

Let $P_{28}(x)$ be the 28th - degree Maclaurin polynomial for f about $x = 2$ where $f(x) = x^{28} + e^{-x/2}$. What is the coefficient of the x^{28} in $P_{28}(x)$?

13. Level: AP4

Let $f(x) = x^2 e^x$. Find the first three terms of the Taylor polynomial for $f(x)$ centered at $x = 1$.



14. Level: AP4

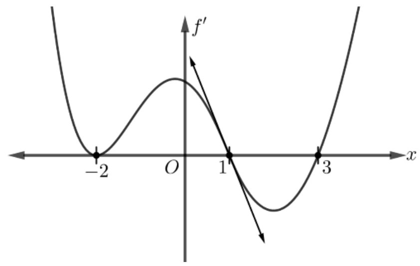
Let $W_3(x) = 2(x-a) + \frac{1}{3!}(x-a)^3$ be the third-degree Taylor polynomial for g about $x = a$, where a is a constant. The graph of g is shown in the figure above. $x = a$ could be the x- coordinate for which of the following points?

(A) Point A

(B) Point B

(C) Point C

(D) Point D



15. Level: AP5

Let f be a function that has derivatives of all orders and let h be a function whose nth - derivative at $x = r$ is given by $h^{(n)}(r) = f^{(n-1)}(r+3)$ for $n \geq 1$. The graph of f', the derivative of f, is shown in the figure above along with the line tangent to the graph of f' at $x = 1$. It is known that the graphs of both f and h pass thru the point $(-2, 3)$. Which of the following could be the third-degree Taylor polynomial for h about $x = -2$?

(A) $y = 3 + 5(x+2) - (x+2)^3$

(B) $y = 3 + 2(x+2) - \frac{5}{4}(x+2)^3$

(C) $y = 3 + \frac{1}{2}(x+2)^2 - \frac{8}{3}(x+2)^3$

(D) $y = 3 + 7(x+2) + \frac{1}{3}(x+2)^2 - 2(x+2)^3$

16. Level: AP3

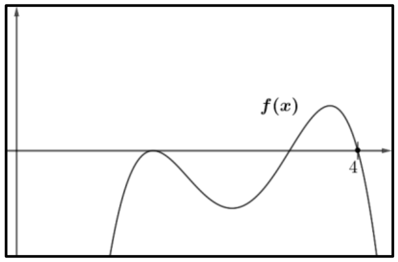
The second-degree Taylor polynomial for h about $x = 3$ is $P_2(x) = -7 + \frac{5}{4}(x-3)^2$. What is the slope of the line tangent to the graph of h at $x = 3$?

(A) -7 (B) 0 (C) $\frac{5}{4}$ (D) $\frac{5}{2}$

17. Level: AP4

Let $P_n(x)$ be the nth - degree Maclaurin polynomial for $k(x) = \cos(2x)$ where $n \geq 0$. What is the coefficient of x^{14} in $P_n(x)$?

(A) $-\frac{1}{14!}$ (B) $\frac{2^{13}}{14!}$ (C) $-\frac{2^{14}}{14!}$ (D) $\frac{2^{14}}{14!}$



18. Level: AP4

Let $P_3(x)$ be the third-degree Taylor polynomial for $f(x)$ about $x = 4$. The graph of $f(x)$ is shown in the figure above. Which of the following could be the coefficient of x^2 in $P_3(x)$?

(A) -3 (B) 0 (C) $\frac{1}{2!}$ (D) 3

19. Level: AP4

Let $P_3(x)$ be the third-degree Taylor polynomial for $f(x)$ about $x = 4$. The graph of $f(x)$ is shown in the figure above. Which of the following could be the constant in $P_3(x)$?

(A) -3 (B) 0 (C) $\frac{1}{2!}$ (D) 3

20. Level: AP4

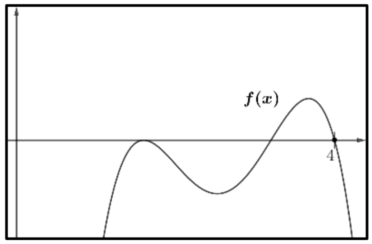
Let $P_2(x)$ be the second-degree Taylor polynomial for $f(x)$ about $x = 4$. The graph of $f(x)$ is shown in the figure above. Which of the following could be the expression for $P_2(x)$?

(A) $4 - 5x - \frac{3x^2}{2!}$

(B) $-5(x-4) - \frac{3(x-4)^2}{2!}$

(C) $5(x-4) - \frac{3(x-4)^2}{2!}$

(D) $4 - 5(x-4) - \frac{3(x-4)^2}{2!}$



21. Level: AP4

Let $P_2(x)$ be the second-degree Taylor polynomial for g about $x = 4$ where $g(x) = 3 - \int_4^x f(t)dt$. The graph of f is shown in the figure above. Which of the following could be the coefficient of x in $P_2(x)$?

(A) -3 (B) $-\frac{1}{2}$ (C) 0 (D) 3

22. Level: AP4

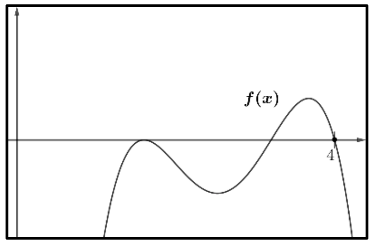
Let $P_4(x)$ be the fourth-degree Taylor polynomial for g about $x = 4$ where $g(x) = 3 - \int_4^x f(t)dt$. The graph of $f(x)$ is shown in the figure above. Which of the following could be the coefficient of x^2 in $P_4(x)$?

(A) -3 (B) $-\frac{1}{2}$ (C) 0 (D) 3

23. Level: AP4

Let $T_2(x)$ be the second-degree Taylor polynomial for h about $x = 2$ where $h(x) = 5x + \int_{x^2}^4 f(t)dt$. The graph of $f(x)$ is shown in the figure above. Which of the following could be the coefficient of x^2 in $T_2(x)$?

(A) -6 (B) 0 (C) 12 (D) There is not enough information given.



24. Level: AP4

Let $Y_2(x)$ be the second-degree Taylor polynomial for h about $x=2$ where $h(x)=5x+\int_{x^2}^{4} f(t)dt$. The graph of $f(x)$ is shown in the figure above. Which of the following could be the expression for $Y_2(x)$?

(A) $20+5(x-2)+\frac{11(x-2)^2}{2!}$

(B) $20+5(x-2)-\frac{11(x-2)^2}{2!}$

(C) $10+5(x-2)+\frac{11(x-2)^2}{2!}$

(D) $10+5(x-4)+\frac{11(x-4)^2}{2!}$

25. Level: AP4

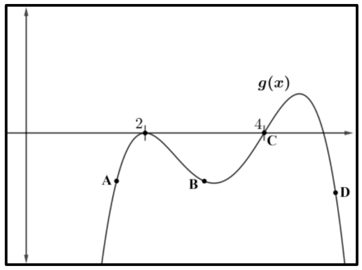
Let $K_6(x)$ be the sixth-degree Taylor polynomial for k about $x=4$ where $k(x)=e^{f(5x-x^2)}$. The graph of $f(x)$ is shown in the figure above. Which of the following could be the equation of the line tangent to graph of $K_6(x)$ at $x=4$?

(A) $y=1+3(x-4)$

(B) $y=1-3(x-4)$

(C) $y=4+3(x-4)$

(D) $y=-4-3(x-4)$



26. Level: AP4

Let $P_3(x)$ be the third-degree Taylor polynomial for $g(x)$ about $x = 2$. The graph of $g(x)$ is shown in the figure above. Which of the following could be the coefficient of x^2 in $P_3(x)$?

(A) -3 (B) 0 (C) $\frac{1}{2!}$ (D) 3

27. Level: AP4

The function g has derivatives of all orders, and a portion of the graph of g is shown in the figure above. Let $P_2(x)$ be the second-degree Taylor polynomial for g about the point B. Which of the following could be the expression for $P_2(x)$?

(A) $-5-2x+3x^2$

(B) $-4-(x-3)+2(x-3)^2$

(C) $-6+(x-3)-\frac{(x-3)^2}{2!}$

(D) $-3-\frac{1}{2}(x+3)+\frac{9}{2!}(x+3)^2$

28. Level: AP4

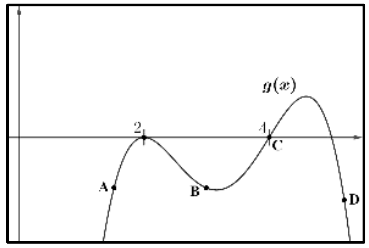
The function g has derivatives of all orders, and a portion of the graph of g is shown in the figure above. Let $T_2(x)$ be the second-degree Taylor polynomial for g about the point C. Which of the following could be the first two terms for $T_2(x)$?

(A) $4+3x$

(B) $-1+2(x-4)$

(C) $3(x-4)$

(D) $3-2(x-4)$



29. Level: AP4

Let $M_2(x) = -1 + 2(x-c) - \frac{3}{4}(x-c)^2$ be the second-degree Taylor polynomial for g about $x = c$, where c is a constant. The graph of g is shown in the figure above. $x = c$ could be the x- coordinate for which of the following points?

(A) Point *A*

(B) Point *B*

(C) Point *C*

(D) Point *D*

30. Level: AP4

Let $H_2(x) = -3 - (x-c) - \frac{1}{2!}(x-c)^2$ be the second-degree Taylor polynomial for g about $x = c$, where c is a constant. If the graph of g is shown in the figure above, then $x = c$ could be the x- coordinate for which of the following points?

(A) Point *A*

(B) Point *B*

(C) Point *C*

(D) Point *D*