"It is better to solve one problem five different ways than to solve five different problems"
- - - Poyla

HOW DO WE "LOOK" AT AP CALCULUS?

Ramblings by Dr. Joseph Brandell, because there was no one there to stop him

Five ways we examine concepts and their analysis in mathematics

There are five ways we examine concepts and their analyses in mathematics: algebraically, geometrically, numerically, verbally and theoretically. The purpose of this discussion is to expound on these learning approaches as they impact AP Calculus.

Algebraically (i.e., symbolically, abstractly, writing and solving equations) – Typically, AP Calculus students like this approach, and are usually pretty good in this area. However, many times it is to the detriment of the other four ways to examine mathematics.

Geometrically (i.e., draw it, graph it, make a picture, see geometric shapes instead of random lines or curves) – Students need to know before they enter Calculus the graphs of several functions, including linear, quadratic, cubic, exponential, log, sine and cosine. They also need to know how to translate the functions, along with "stretching and shrinking" the functions. If these are not mastered prior to coming to your class, students can still achieve, but they will have to learn these "on the run." Over the years with all the graphing technology that is available, students have gotten much better at graphing functions and being able to "know what they are seeing." They must look for geometric connections when given a diagram. Many times, calculus students have gotten by on their ability to "abstract" a problem (heck, they will even try to turn a triangle or a semi-circle into equations in order to abstract the problem instead of looking at the situation geometrically). In general, Calculus students are decent in this area, but they need every opportunity to practice this learning approach in a meaningful way.

Numerically (i.e., patterns in numbers or terms, tables of values) – It is at this juncture where Calculus students are typically weak. The areas of sequences, series, polar functions and trig functions can be problematic if a pattern is not identified. Opportunities for students to develop a sense for numerical patterns might involve inspecting slopes of tangent lines at multiple points on the same curve, finding the general term for an infinite series, and finding specific values for trig and polar functions. It is essential calculus instructors take advantage of these opportunities to enrich this learning style that is typically neglected, and provide ample opportunities "sans calculator" to force the issue.

Verbally (e.g., intuitively, "let's talk about it," "let's write about it," "justify your answer") – Students have shown much growth in this area over the past few years, but we still have a ways to go. Whenever you can get your students to clearly explain their work, it is a good day. **We must get the kids to know that simply citing a definition is NOT the same as justifying their answer.** We must get the kids to "personalize" the definition to fit or apply to their problem specifically. This method of learning is still "work in progress" for many of our Calculus students.

Theoretically – In the AP Course Outline, we really don't emphasize the theoretical justification too much. It is not really on the AP Exam formally. Yet, we really do our advanced students a disservice if we don't at least take "theoretical" opportunities when they present themselves. The two opportunities to formally examine the theory behind the concepts is the definition of the derivative, and the Fundamental Theorem of Calculus. Informally, other concepts to examine theoretically include the Mean Value Theorem for Derivatives, the Mean Value Theorem for Integrals and average value. Typically, these concepts are best explored in the context of explaining other applications (Authors note: Ok, the definition of the derivative is not really a theoretical approach to derivatives, but who wants to do epsilon-delta proofs? Not me! Once I got my PhD, I made a promise to myself that I will never do another epsilon-delta proof again, and I will keep this promise!)

Spiral until you get dizzy

Each topic does not have to be taught "within an inch of its life" for every test. When this is done, many times those topics are not revisited until the final exam or the AP Exam. An approach that has been helpful is to literally assess each topic at three levels: exposure, general mastery, and AP mastery. Allow formal exams to have several topics assessed at various levels of mastery, almost making every exam a comprehensive final. On the surface, this strategy might "feel" like it would slow down your curriculum pacing, but it can really help a class go faster through material, and the students obtain a deeper level of understanding in the process.

Embed, overlap, compare/contrast and connect as many topics as possible

Always, always, always look for ways to connect material. Use memory prompts, weird songs and "stupid" patterns to help the kids memorize concepts (the kids think they are stupid, but they use them). In addition, teach "complementing" concepts together, like derivatives and antiderivatives, antiderivatives and slope fields, and area bound by polar curves with area bound by Cartesian equations. This approach really helps the students make connections so they don't feel like they have a million isolated topics to learn.

Make an "AP moment"

Spend some time looking at the last 5 or 6 years of free-response questions from AP Exams. Find problems that fit your content, and determine when you can introduce those particular problems. Don't wait until three weeks before the AP Exam to show your students "AP style" problems; if you do they will freak out. The more we can show them exam questions, the more we can honestly say taking the exam will be "no big deal." Besides, once you have several "AP moments" in your class, you will have enough material for sub plans so you can go to an AP workshop.

Technology: Friend or Foe?

The use of technology in today's mathematics classroom can really help the students "see" concepts both geometrically and numerically. In particular, the concepts of sequences and series, approximating a function at a point with a tangent line, and using the function integrator to find arc length virtually "come to life" with graphing technology. That said, just be careful. It is obvious to Mathematics Educators that students need to have "pencil and paper" skills before they should be able to use a calculator. The AP Exam provides a clear model of what students need to be able to do by hand and with a calculator.

There is life after the AP Exam . . . and some of it is intelligent!

Authors reflection: I have done several things after the AP Exam with students: save my last chapter's assessments until after the AP Exam (the scores were horrible), do nothing (administration really loved that one), continued with new material (the students really loved that one), and had them tutor freshman in Algebra classes. What I do now is have my students create an "instructional tool" that can be used in future classes. Students have the latitude to make a video, parody a music video (for the new teachers, these used to be on MTV), a tutorial CD or DVD, a children's book, a calculator program or any other tool that could be used to support the teaching of some area of Calculus. The kids love it, and I use these projects throughout the year to assist in the instruction of the topics.

That's about it . . . I have rambled on too long as it is . . . I welcome any feedback from you on these topics. You have my email address at the bottom. If you have any questions, agreements, disagreements, additions or corrections, please let me know. I tell people teaching is "selective thievery" and I always look forward to stealing other people's great ideas (err, I mean sharing other peoples ideas) in workshops and in the classroom.

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BC Calculus – Final Project

Objective: To design, create, and present an instructional tool for AP Calculus
Points Possible: 70
Date Due: Tuesday, May 20, 2014

Project Description, Guidelines, and Timelines

Phase One – Project Proposal (10 points)
In this phase, students will propose the topic to be explored, group member(s) and the role for each group member. Typically, only four members are allowed in a group. Groups can have members from both second and fourth period. For acceptance of the proposal, an outline of the overall project is described in *some* detail, along with the individual responsibilities of each group member. Each group will choose a different area of study, so proposal acceptance is on a first-come, first-serve basis. It is essential the groups begin this activity early, as acceptance of the proposal can be lengthy. Examples of projects developed in previous years include music videos, children's books, audiotapes, instructional guides, "Jeopardy" games for the AP exam and parodies of movies and television shows. In addition, you (and/or a group) might prefer to complete a project where you will be a consultant for an oil company. You will develop and present a proposal on the feasibility of drilling for oil on a sector of land. Details to the project will be provided. The group that presents the best feasibility study, determined by the president of the oil company, will be given gas cards.
Date Due – Thursday, April 18, 2014

Phase Two – Project Refinement (20 points)
In this phase, the students submit the detailed outline of their project. This document includes the areas of responsibility for each group member, along with a script of the activity. Though the script may be modified somewhat after this date, the main ideas and content are provided at this time.
Date Due – Friday, May 9, 2014

Phase Three – Project completion (30 points)
ALL projects must be submitted on time. The grading rubric will follow the "5 C's" as determined by Ms. Fortunate:

- Correct
- Creative
- Cute
- Complete
- Clean

Though all of these "C's" are important, in an Orwellian way, some "C's" are more equal than others (like the last one). Be creative and have fun with this, but don't be inappropriate!
Date Due – Tuesday, May 20, 2014

Phase Four – Learner Log / Reflection Journal (10 points)
Each individual student will keep a detailed journal of his/her activities. All entries must include the date of the activity, what was done during that time, who was present (if more than one person is in the activity) and what was learned/determined from that session. LL/RJ is assessed similarly to other notebook checks. The final entry into the journal will be an assessment of the project itself and student benefits, an assessment of the group members and a self-assessment. Provide detail to your entries. Weak entries will correspond to a weak score in this phase.
Date Due – Tuesday, May 20, 2014

Oral Presentation Rubric

TRAIT	4	3	2	1
NONVERBAL SKILLS				
EYE CONTACT	Holds attention of entire audience with the use of direct eye contact, seldom looking at notes.	Consistent use of direct eye contact with audience, but still returns to notes.	Displayed minimal eye contact with audience, while reading mostly from the notes.	No eye contact with audience, as entire report is read from notes.
BODY LANGUAGE	Movements seem fluid and help the audience visualize.	Made movements or gestures that enhances articulation.	Very little movement or descriptive gestures.	No movement or descriptive gestures.
POISE	Student displays relaxed, self-confident nature about self, with no mistakes.	Makes minor mistakes, but quickly recovers from them; displays little or no tension.	Displays mild tension; has trouble recovering from mistakes.	Tension and nervousness is obvious; has trouble recovering from mistakes.

COMMENTS:

VERBAL SKILLS	5	4	2	1
ENTHUSIASM	Demonstrates a strong, positive feeling about topic during entire presentation.	Occasionally shows positive feelings about topic.	Shows some negativity toward topic presented.	Shows absolutely no interest in topic presented.
ELOCUTION	Student uses a clear voice and correct, precise pronunciation of terms so that all audience members can hear presentation.	Student's voice is clear. Student pronounces most words correctly. Most audience members can hear presentation.	Student's voice is low. Student incorrectly pronounces terms. Audience members have difficulty hearing presentation.	Student mumbles, incorrectly pronounces terms, and speaks too quietly for a majority of students to hear.

COMMENTS:

CONTENT	6	5	3	1
SUBJECT KNOWLEDGE	Student demonstrates full knowledge by answering all class questions with explanations and elaboration.	Student is at ease with expected answers to all questions, without elaboration.	Student is uncomfortable with information and is able to answer only rudimentary questions.	Student does not have grasp of information; student cannot answer questions about subject.
ORGANIZATION	Student presents information in logical, interesting sequence which audience can follow.	Student presents information in logical sequence which audience can follow.	Audience has difficulty following presentation because student jumps around.	Audience cannot understand presentation because there is no sequence of information.
MECHANICS	Presentation has no misspellings or grammatical errors.	Presentation has no more than two misspellings and/or grammatical errors.	Presentation has three misspellings and/or grammatical errors.	Student's presentation has four or more spelling and/or grammatical errors.

COMMENTS:

BC Calculus

"Finally, we get to the crux of my problem!"

OK . . . we have examined binomial expansion and utilized Pascal's Triangle to determine the coefficients. We have connected, as did Sir Isaac Newton, derivatives and antiderivatives with the coefficients of the expanded binomials.

When we wanted to show a binomial to a power was the same as the expanded binomial, we used two ways. First, we graphed both the expressions and showed how they mapped onto each other. We also used a table of values to correlate the values for each expression at common x-values. This seemed to be sufficient . . . for now.

I am going to give you two different binomials to a power. I want you to build the expanded polynomial using the derivative/antiderivative process we developed in class. To verify that your polynomials are actually the same as the original binomials to a power, I want you to sketch each of the expressions in the given window. This is the same process we utilized in class. Let your viewing window be X: {-3, 3} and Y: {-2, 2}. I then want you do make a concluding statement about what you noticed about the expressions, the graphs, and how they relate.

In review, this is what I want for each binomial to a power:

- α the expanded polynomial determined by the derivative/antiderivative process;
- α a sketch of each of the expressions on the same viewing window, with each of the functions labeled; and,
- α a written explanation/justification of what is happening with the two curves.

Problem 1: $(1-x)^{-1}$

The "Crux", Taylor Polynomials, and the "Big Five"

Here is when we put it all together!

We discussed how we could use a series to represent different functions. There was a demonstration that showed the binomial expansion of $(1-x)^n$ and how it the actual expansion equaled the binomial expression. In general, binomial expansion works as follows:

$$(x+y)^n = \sum_{k=0}^{n} \frac{n!}{(n-k)!*k!} x^{n-k} * y^k = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \ldots + \frac{n!}{(n-k)!k!}x^{n-k}y^k + \ldots$$

At this point, we examined what would happen to $(1-x)^n$ when $n=-1$. We expanded the binomial according to the Binomial Theorem and reflected upon what would happen to the graphs of the binomial and the expansion. **The first assignment was to see how $(1-x)^{-1}$ fit on the same graph with $1 + x + x^2 + x^3 + x^4 + x^5 + \ldots$ and speculate or reflect as to the reason why.**

Next, we explore the concept of a general term (sometimes call the "nth" term). This is the summation representation for all terms in the series. For example, we saw how the series $1 + x + x^2 + x^3 + x^4 + x^5 + \ldots$ could represent $\frac{1}{1-x}$ over some interval. We also could determine the general term as x^n. Thus, we write the following:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \ldots + x^n + \ldots$$

Notice the " . . . " after the general term. This is an important feature, as it shows the series goes on infinitely even after the "nth" term.

All of these functions are centered (or based around) x = 0. This is a key aspect to the problem. If the center changes, so does the equation. We noticed that the series, upon adding terms, mapped more closely to $\frac{1}{1-x}$ over the interval (-1, 1). This interval is known as the interval of convergence (aka, IOC).

Four other functions, along with their series, are provided for analysis and examination. These five functions: $\sin x, \cos x, e^x, \frac{1}{1-x}, \frac{1}{1+x}$, are referred to as the "big five." **These five functions are written as being centered at x = 0. This is very important to remember! If the center is ever something other than zero, you will have to build the Taylor polynomials (ominous voice please) "term by term."**

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 + \ldots + (-1)^n \cdot x^n + \ldots.$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \ldots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + \frac{(-1)^n \cdot x^{2n+1}}{(2n+1)!} + \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

The IOC for $\frac{1}{1-x}$ and $\frac{1}{1+x}$ is (-1, 1). However, the IOC for sin x, cos x and e^x is all real numbers (i.e., $x \in \Re$ or $(-\infty, \infty)$). So cool! We will explore this rationale second semester (as I still have to have my fun!).

As long as the center is x = 0, you can use these polynomials as they are written. I advise you to memorize them, as it will save you time from having to build them. Of the five, the Taylor polynomials, centered at x = 0, for sin x, cos x and e^x are the most important to memorize first. Finally, a Taylor series (or a Taylor polynomial) centered at x = 0 is often called a Maclaurin series (or a Maclaurin polynomial).

BC Calculus – Introduction to Infinite Series ("Embedded Unit")

Essential Questions

What does it mean to approximate? Why should we approximate anything if we can find the "exact" representation?

Enduring Understanding

Students will make connections between an infinite series and a function, and value the need to approximate a function as an infinite polynomial.

Students will be able to:

- Differentiate an infinite series and determine its general term
- Integrate an infinite series and determine its general term
- Perform standards operations (+, -, *, /) on an infinite series
- Substitute functions into an infinite series
- Approximate a function value using the infinite series approximation

Students will know:

- The relationship between a function and its polynomial approximation
- The rationale for writing a function as an infinite series
- Any function can be approximated by an infinite series (infinite polynomial)

Assessments

Group Project (performance tasks and individual requirements):
In groups of four, students will be given a different aspect of infinite Maclaurin series to examine that ties into prior knowledge of derivatives, integrals and functions. Each group will execute the following tasks:

- Formulate the strategy needed to present this extension of prior knowledge;
- Determine meaningful sample problems for the class to complete during guided practice;
- Establish two homework problems for each portion of their content;
- Create a 20-minute lesson, which includes demonstration and guided practice, that involves all group members in a meaningful way;
- Create an answer key for the class to check their assigned homework;
- Provide copies of the answer key to all students in the class; and,
- Write a reflection examining the following issues:
    - Your understanding of the content for which you were responsible;
    - The value of completing this assignment;
    - An evaluation of your group members.

Rubric for each of the assessments is cited at the end of this section

Elements of the Project

Each group will present a different aspect of sequences and series as it applies to our study of calculus. The order of the topics to be presented is:

- Review of geometric series and finding the general term;
- The derivative of a function is the derivative of the series of the function (basic functions);
- The product or quotient rules apply to derivatives of series representations;
- Series can be used to approximate function values;
- Nested polynomial functions in the "big five" can be differentiated as a series in the same fashion;
- The integral of a function is the integral of the series of the function;
- Convergence of series is similar to that of geometric series; and,
- Review if linear approximations (tangent lines).

Analysis of Project Elements

Review of geometric series and finding the general term

Students will be able to:

- Write a given series from summation notation;
- Write summation notation from a given series; and,
- Determine if a geometric series converges and if so, give its sum.

The presenters first will review the series approximation for the "big five" functions: $Sin\,x,\ Cos\,x,\ e^x,\ \frac{1}{1-x},\ \frac{1}{1+x}$. Particular attention will be paid to the generation of the general term and summation notation. The presenters will provide to the whole group several examples of series in expanded form for the whole group to put in summation notation. When providing a numeric series, students will determine if the series is geometric or not, and if so, does it converge or diverge. If the geometric series converges, then the sum should be determined.

Sample problems:

Write in summation notation and find the general term for: $x+\frac{x^2}{2}+\frac{x^3}{4}+\frac{x^4}{8}+\ldots$

Expand to its first five terms and give its general term: $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(2x)^n}{n}$; $\sum_{n=0}^{\infty}\left(\frac{4}{3}\right)^2\left(\frac{3}{8}\right)^{n+1}$

Tell whether the series converges or diverges; if it converges, give its sum:

$1+\frac{2}{3}+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^3+\ldots+\left(\frac{2}{3}\right)^n+\ldots$ $\qquad$ $\frac{5}{3}-\frac{15}{6}+\frac{45}{12}-\frac{135}{24}+\ldots$

Presenters will make-up two problems each as homework for their class.

Reference pages 457-460, and page 466 of text (Finney, Demana, Waits, Kennedy) for additional support

Group 3

A Taylor Polynomial is the polynomial approximation for any function.
Students will be able to:

- Use the general format for a Taylor Polynomial, which is in the form:

$$y=\frac{f^{(0)}(x_1)(x-x_1)^0}{0!}+\frac{f'(x_1)(x-x_1)^1}{1!}+\frac{f''(x_1)(x-x_1)^2}{2!}+\ldots+\frac{f^{(n)}(x-x_1)^n}{n!}+\ldots.$$

Students will note that a tangent can be written in the form $y=m(x-x_1)+y_1$ is the same as writing the function in the form $y=y_1+f'(x_1)(x-x_1)$, which can be transferred into the following equation: $y=\frac{f^{(0)}(x_1)(x-x_1)}{0!}+\frac{f'(x_1)(x-x_1)}{1!}$. This is called a "first order Taylor Polynomial, centered at x = 1." Notice each term in the Taylor Polynomial has three parts and is in the form: $\frac{\text{[derivative part] * [function part]}}{\text{[factorial part]}}$. Each Taylor Polynomial is "centered" at the point $(f(x_1),x_1)$, which is treated like a point of tangency. Students should create four problems for homework (one problem apiece) that reflect this phenomenon. In the mix, students should try to approximate a function using a higher order than the original polynomial.

<u>Example</u>: Write a third order Taylor Polynomial, centered at x = 1, to approximate the curve $y=x^4-6x^3+4x^2-3x+5$; use this Taylor Polynomial to approximate $y(1.1)$.

<u>Solution</u>:
Zero order: $f(x)=y=x^4-6x^3+4x^2-3x+5$; $f(1)=1$
First order: $f'(x)=4x^3-18x^2+8x-3$; $f'(1)=-9$
Second order: $f''(x)=12x^2-36x+8$; $f''(1)=-18$
Third order: $f^{(3)}(x)=24x-36$; $f^{(3)}(1)=-12$

Build the Taylor Polynomial, $T(x)$, "term by term" and each term "part by part":

$$f(x)\approx T_3(x)=\frac{f(1)\cdot(x-1)^0}{0!}+\frac{f'(1)\cdot(x-1)^1}{1!}+\frac{f''(1)\cdot(x-1)^2}{2!}+\frac{f^{(3)}(1)\cdot(x-1)^3}{3!}$$

$=1+-9(x-1)+-9(x-1)^2+2(x-1)^3$;

$f(1.1)\approx T_3(1.1)=1-9(.1)-9(.1)^2+2(.1)^3=1-.9-.09+.002=0.012$

Version 09.02.13

Guidelines for *Retro Points*

Retro Points are points that are earned by demonstrating you have mastered skills that were not mastered on previously given tests or quizzes.

Eligibility for earning *Retro Points:*

- Must score lower than a designated threshold for mastery (typically 80%) established before a formal assessment; if there are two parts to a formal assessment (e.g., calculator-active and non-calculator), the threshold MAY BE determined by the combined total of the two parts, or each part treated separately.
- There must be a specific content objective that was not mastered; losing points due to arithmetic or algebraic mistakes is not necessarily a specific content objective.
- All CORE problems assigned in the unit guide are completed prior to taking the assessment.

Process for earning *Retro Points:*

- Identify the specific content objectives that were not mastered on the assessment
- Rework the problems that were wrong on the assessment that reflect those specific content objectives
- Complete all SUPPORT problems assigned in unit guide.
- Contact Dr. Brandell to individually show mastery of the material
- Work with Dr. Brandell to determine what assessments will reflect the level of mastery on those specific content objectives that is required for earning *Retro Points*
- Demonstrate the appropriate level of mastery for the concepts on the suitable assessment(s)
- Bring the following papers to Dr. Brandell:
    - *Retro Points* Checklist with Dr. Brandell's initials
    - Previous assessment showing the specific content objectives that were not mastered
    - Recent assessment showing mastery of specific content objectives

Retro Points
Checklist

____
Name

______________________ Student is eligible to earn *Retro Points*

______________________ Title and date of assessment(s) where specific content objectives were not achieved

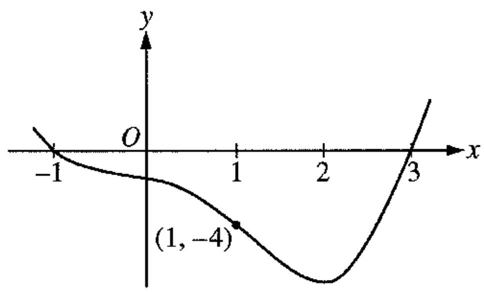
______________________ Specific content objective(s) not mastered

______________________ Worked with teacher and has shown mastery

______________________ Determine assessment(s) that will earn *Retro Points*

______________________ Demonstrate mastery on the designated assessments

______________________ Attach the assessments with this checklist and submit to teacher



Graph of f'

5. Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.

   (a) Write an equation for the line tangent to the graph of g at $x = 1$.

   (b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

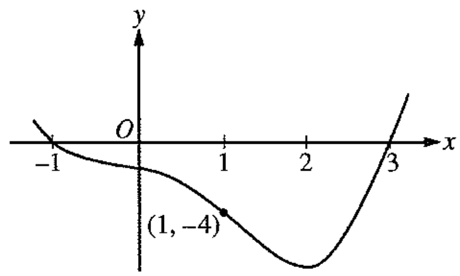
   (c) The second derivative of g is $g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.

   (d) Find the average rate of change of g', the derivative of g, over the interval $[1, 3]$.

WRITE ALL WORK IN THE EXAM BOOKLET.

Question 5

Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.



Graph of f'

(a) Write an equation for the line tangent to the graph of g at $x = 1$.

(b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

(c) The second derivative of g is $g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.

(d) Find the average rate of change of g', the derivative of g, over the interval $[1, 3]$.

(a) $g(1) = e^{f(1)} = e^2$

$g'(x) = e^{f(x)} f'(x), \quad g'(1) = e^{f(1)} f'(1) = -4e^2$

The tangent line is given by $y = e^2 - 4e^2(x - 1)$.

$3 : \begin{cases} 1 : g'(x) \\ 1 : g(1) \text{ and } g'(1) \\ 1 : \text{tangent line equation} \end{cases}$

(b) $g'(x) = e^{f(x)} f'(x)$

$e^{f(x)} > 0$ for all x

So, g' changes from positive to negative only when f' changes from positive to negative. This occurs at $x = -1$ only. Thus, g has a local maximum at $x = -1$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(c) $g''(-1) = e^{f(-1)}\left[(f'(-1))^2 + f''(-1)\right]$

$e^{f(-1)} > 0$ and $f'(-1) = 0$

Since f' is decreasing on a neighborhood of -1, $f''(-1) < 0$. Therefore, $g''(-1) < 0$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(d) $\dfrac{g'(3) - g'(1)}{3 - 1} = \dfrac{e^{f(3)} f'(3) - e^{f(1)} f'(1)}{2} = 2e^2$

$2 : \begin{cases} 1 : \text{difference quotient} \\ 1 : \text{answer} \end{cases}$

B-Squared

12. Let $\frac{dy}{dx} = x + y$

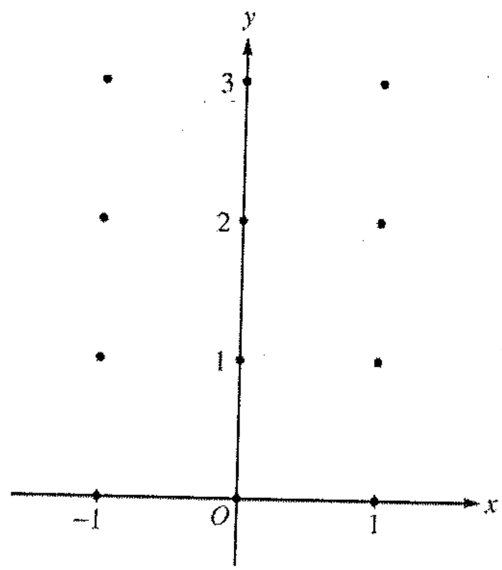
(a) Suppose the point (2, 3) is a point on a particular solution to $\frac{dy}{dx}$. What is the equation of the tangent line to the particular equation at that point?

(b) Find $\frac{d^2y}{dx^2}$

(c) Find $\frac{d^{213}y}{dx^{213}}$ at the point (5, 1)

Consider the differential equation $\frac{dy}{dx} = x^2 y$

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the pink test booklet.)**



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 1$.