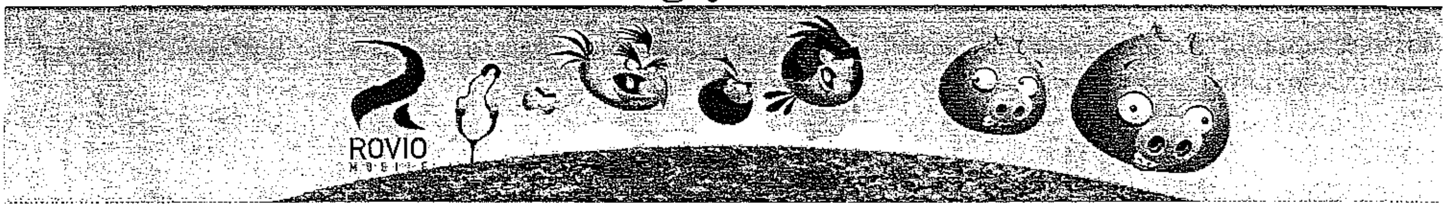
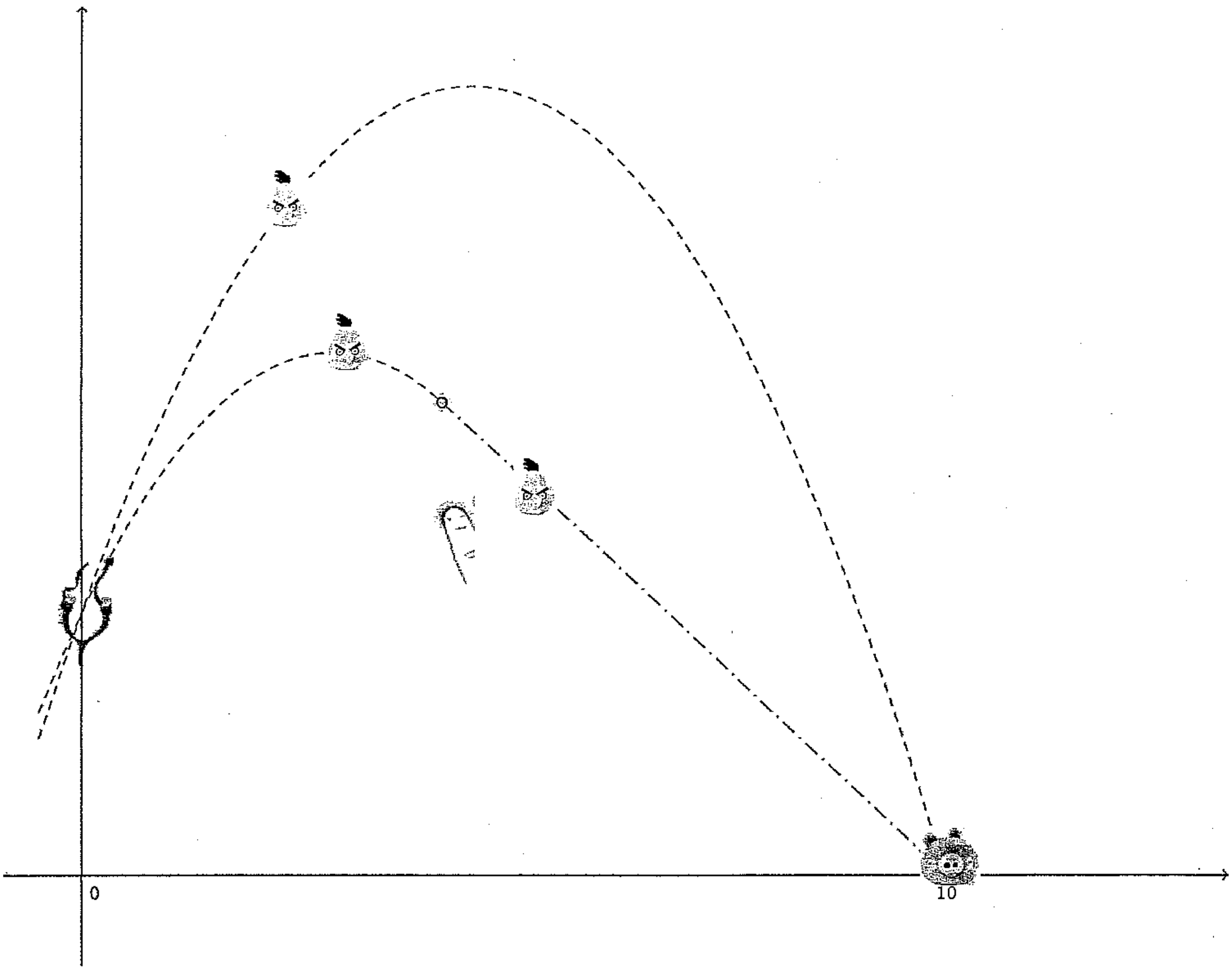
Angry Birds





When the **Yellow Bird** is released from the slingshot at the point $(0,3)$, it travels in a parabolic path. If the screen is tapped at some time $t > 0$, the yellow bird stops traveling along the parabola and continues along a linear path determined by position when the screen is tapped. The object of the game is to hit the pig, at $(10,0)$, that stole the eggs.

Project:
Model the motion of yellow bird. There are three levels. Complete all or one level. Each level requires you to give algebraic, tabular and graphical evidence that your model works.

Level 1

Write an equation for a parabola that models a path that yellow bird takes to strike the pig without a finger tap using the positions given above.

Level 2

Write a different equation for a parabola that models a path that yellow bird takes to strike the pig with a finger tap using the positions given above.

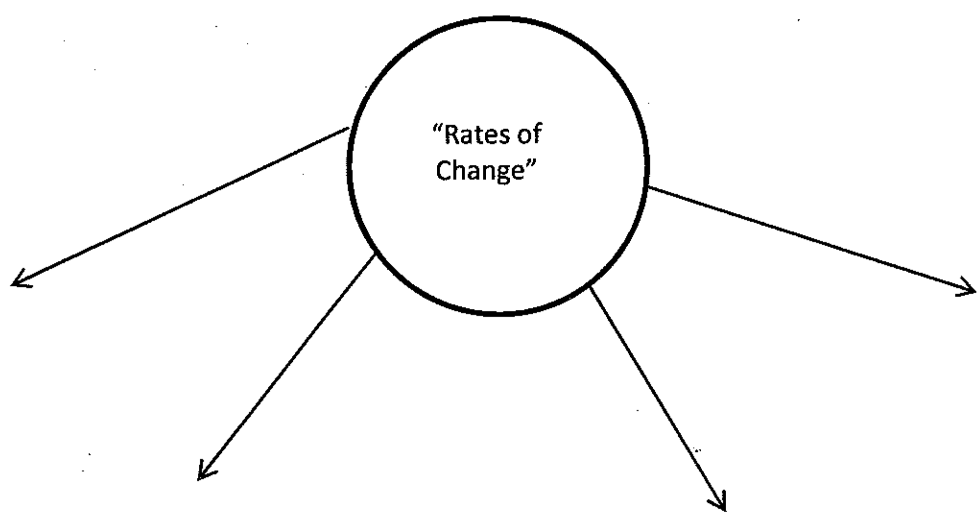
Level 3

Write an equation for a parabola that models a path that yellow bird takes to strike the pig with a finger tap where the positions of the slingshot and pig are not given. Be sure to consider the pig at a position not on the x-axis.

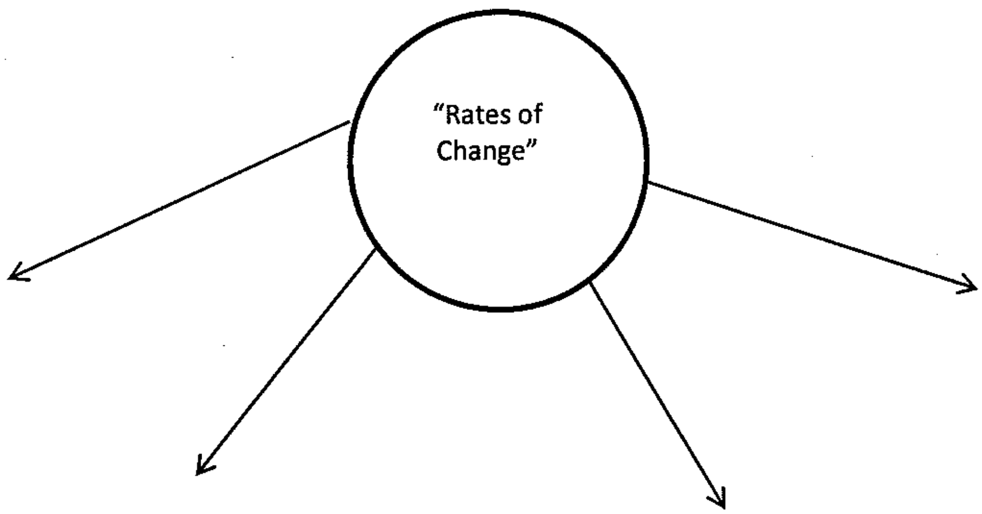
B^2 Secrets Revealed: The Web of $\frac{dy}{dx}$

What does mean for a function to have a "rate of change?"

How do we denote "rates of change?"



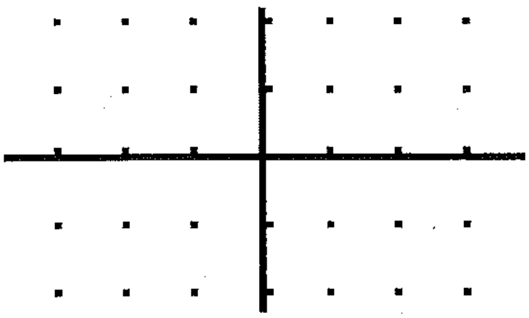
How do we use "rates of change?"
For what processes are "rates of change" used?



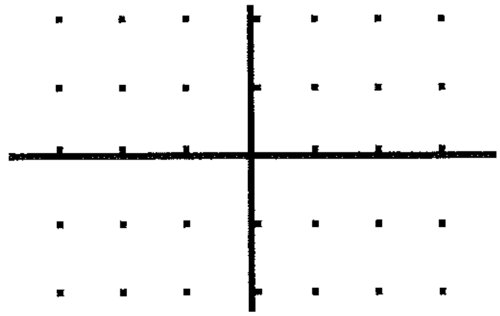
SLOPE FIELDS

Draw a slope field for each of the following differential equations.

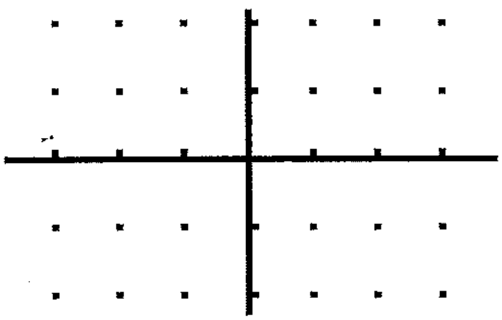
1. $\frac{dy}{dx} = x+1$



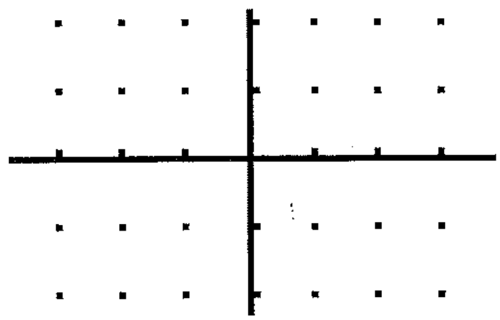
2. $\frac{dy}{dx} = 2y$



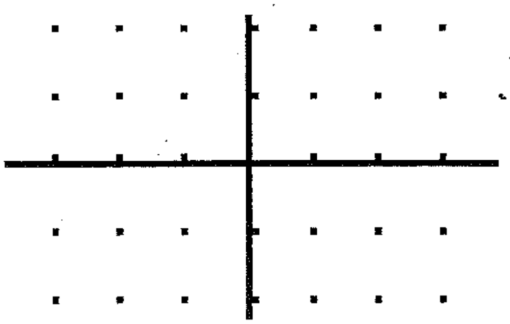
3. $\frac{dy}{dx} = x+y$



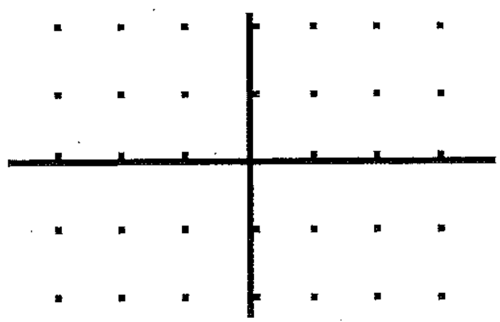
4. $\frac{dy}{dx} = 2x$



5. $\frac{dy}{dx} = y-1$

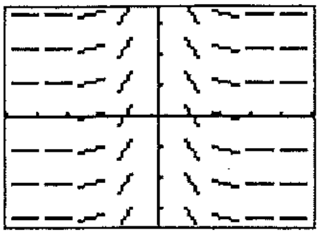


6. $\frac{dy}{dx} = -\frac{y}{x}$

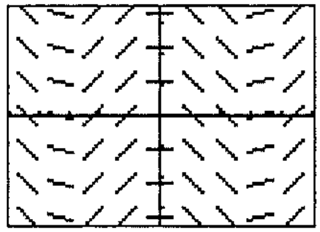


Match each slope field with the equation that the slope field could represent.

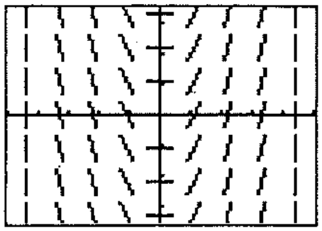
(A)



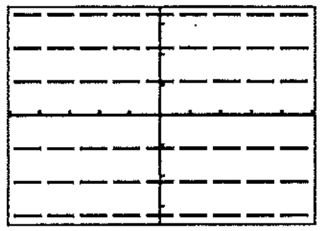
(B)



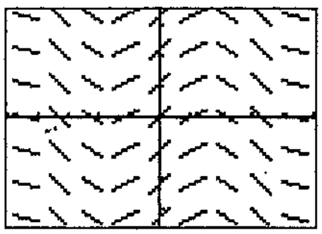
(C)



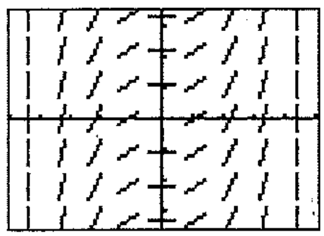
(D)



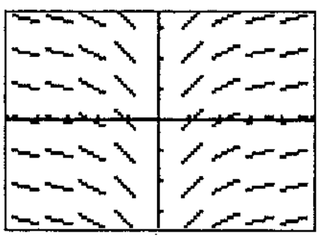
(E)



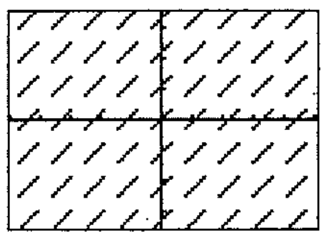
(F)



(G)



(H)



7. $y = 1$

8. $y = x$

9. $y = x^2$

10. $y = \frac{1}{6}x^3$

11. $y = \frac{1}{x^2}$

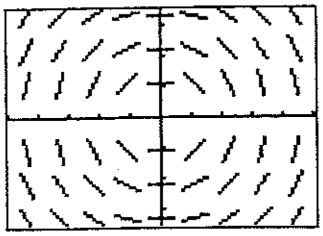
12. $y = \sin x$

13. $y = \cos x$

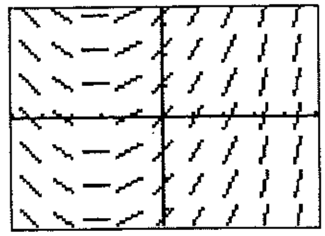
14. $y = \ln|x|$

Match the slope fields with their differential equations.

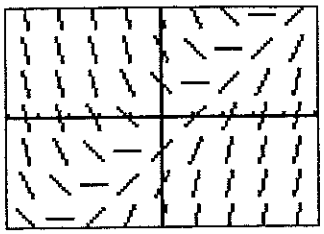
(A)



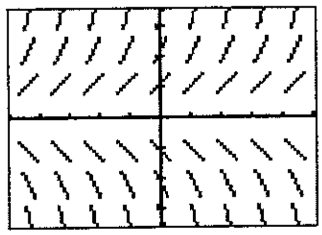
(B)



(C)



(D)



15. $\frac{dy}{dx}=\frac{1}{2}x+1$

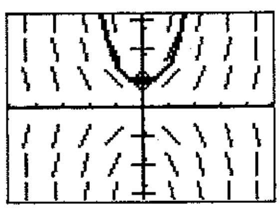
16. $\frac{dy}{dx}=y$

17. $\frac{dy}{dx}=x-y$

18. $\frac{dy}{dx}=-\frac{x}{y}$

---

19. The calculator drawn slope field for the differential equation $\frac{dy}{dx}=xy$ is shown in the figure below. The solution curve passing through the point (0, 1) is also shown.
    (a) Sketch the solution curve through the point (0, 2).
    (b) Sketch the solution curve through the point $(0,-1)$.



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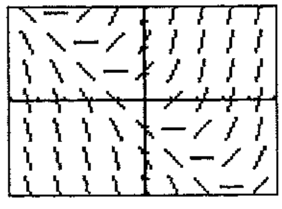
20. The calculator drawn slope field for the differential equation $\frac{dy}{dx}=x+y$ is shown in the figure below.
    (a) Sketch the solution curve through the point (0, 1).
    (b) Sketch the solution curve through the point $(-3, 0)$.



Euler's Method for Approximating a Function (linearization)

$$L(x) = y(x_0) + y'(x_0)(x - x_0) \quad \text{or} \quad L(x) = y_0 + f'(x_0, y_0)(x - x_0)$$

Teaching point: Don't let the student make a bigger deal out of this concept than it is. You are simply finding ordered pairs. You have been given a starting point, a differential and Δx that is constant. Heck, in terms of finding ordered pairs, you are half-way home! Now, we must establish our Δy, which unfortunately changes with each ordered pair. In addition, we are going to have to approximate Δy with dy. But don't worry! Just follow the systematic algorithm outlined below. It is really "no big deal."

Algorithm:

1) Establish $x_0, y_0, \Delta x, \frac{dy}{dx}$ $(or\ y')$

2) Find $\frac{dy}{dx}$ or y' at that point (i.e., $y'(x_0, y_0)$ or $\frac{dy}{dx}|_{(x_0,y_0)}$)

3) Note: Since $y_{n+1} = y_n + \Delta y$, we will use the differential dy to approximate Δy. In a similar fashion, we will approximate dy such that $dy \approx \frac{dy}{dx} * \Delta x$. In general, $y_{n+1} = y_n + \frac{dy}{dx}|_{(x_n,y_n)} * \Delta x$. Specifically, $y_1 = y_0 + \frac{dy}{dx}|_{(x_0,y_0)} * \Delta x$.

4) Find each part of the "next" ordered pair separately:

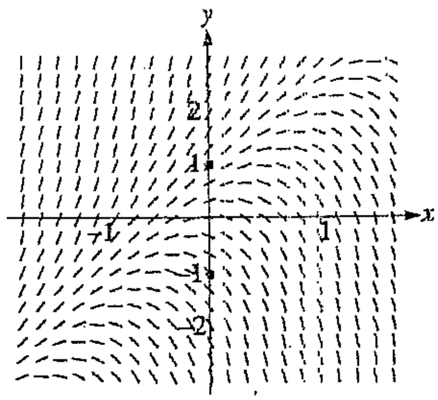
$$x_1 = x_0 + \Delta x$$

$$y_1 = y_0 + dy = y_0 + \frac{dy}{dx}|_{(x_0,y_0)} * \Delta x$$

5) Repeat the process until you have satisfied the conditions of the problem.

5. Consider the differential equation $\frac{dy}{dx} = 2y - 4x$.

(a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point $(0, 1)$ and sketch the solution curve that passes through the point $(0, -1)$.
(Note: Use the slope field provided in the pink test booklet.)



(b) Let f be the function that satisfies the given differential equation with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with a step size of 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

(c) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.

(d) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point $(0, 0)$? If so, is the point a local maximum or a local minimum? Justify your answer.

---

6. The Maclaurin series for the function f is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{(2x)^{n+1}}{n+1} = 2x + \frac{4x^2}{2} + \frac{8x^3}{3} + \frac{16x^4}{4} + \cdots + \frac{(2x)^{n+1}}{n+1} + \cdots$$

on its interval of convergence.

(a) Find the interval of convergence of the Maclaurin series for f. Justify your answer.

(b) Find the first four terms and the general term for the Maclaurin series for $f'(x)$.

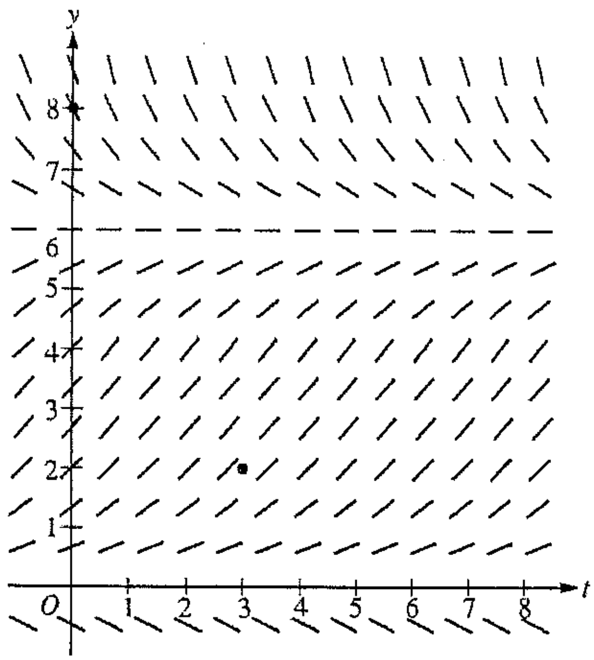
(c) Use the Maclaurin series you found in part (b) to find the value of $f'\left(-\frac{1}{3}\right)$.

END OF EXAMINATION

6. Consider the logistic differential equation $\frac{dy}{dt} = \frac{y}{8}(6 - y)$. Let $y = f(t)$ be the particular solution to the differential equation with $f(0) = 8$.

(a) A slope field for this differential equation is given below. Sketch possible solution curves through the points $(3, 2)$ and $(0, 8)$.

(Note: Use the axes provided in the exam booklet.)



(b) Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$.

(c) Write the second-degree Taylor polynomial for f about $t = 0$, and use it to approximate $f(1)$.

(d) What is the range of f for $t \geq 0$?

WRITE ALL WORK IN THE PINK EXAM BOOKLET.

END OF EXAM

Version 09.02.13

Guidelines for *Retro Points*

Retro Points are points that are earned by demonstrating you have mastered skills that were not mastered on previously given tests or quizzes.

Eligibility for earning *Retro Points*:

- Must score lower than a designated threshold for mastery (typically 80%) established before a formal assessment; if there are two parts to a formal assessment (e.g., calculator-active and non-calculator), the threshold MAY BE determined by the combined total of the two parts, or each part treated separately.
- There must be a specific content objective that was not mastered; losing points due to arithmetic or algebraic mistakes is not necessarily a specific content objective.
- All CORE problems assigned in the unit guide are completed prior to taking the assessment.

Process for earning *Retro Points*:

- Identify the specific content objectives that were not mastered on the assessment
- Rework the problems that were wrong on the assessment that reflect those specific content objectives
- Complete all SUPPORT problems assigned in unit guide.
- Contact Dr. Brandell to individually show mastery of the material
- Work with Dr. Brandell to determine what assessments will reflect the level of mastery on those specific content objectives that is required for earning *Retro Points*
- Demonstrate the appropriate level of mastery for the concepts on the suitable assessment(s)
- Bring the following papers to Dr. Brandell:
  - *Retro Points* Checklist with Dr. Brandell's initials
  - Previous assessment showing the specific content objectives that were not mastered
  - Recent assessment showing mastery of specific content objectives

Retro Points
Checklist

____
Name

______________________ Student is eligible to earn *Retro Points*

______________________ Title and date of assessment(s) where specific content objectives were not achieved

______________________ Specific content objective(s) not mastered

______________________ Worked with teacher and has shown mastery

______________________ Determine assessment(s) that will earn *Retro Points*

______________________ Demonstrate mastery on the designated assessments

______________________ Attach the assessments with this checklist and submit to teacher

Conventions

The purposes and methods of scoring the AP Calculus Exam are different from the purposes and methods we, as calculus teachers, use with our own students. Some of the conventions are used to ensure consistency and accuracy, as well as fairness to the student. We attempt to give each student credit when the student has shown knowledge and understanding of calculus. There are several accepted practices that pertain to reading AP Calculus Exams. These include the following.

- *Copy errors* – Typically a one-point deduction is taken for a copy error, and you should continue to read the student's work for correctness from that point forward. A student may need to meet eligibility requirements in order to receive further credit.
- *Saying too much* – When a student has solved a problem or answered a question completely, but then goes on and does additional work, which may be incorrect, the student has said *too much*. Depending on the question, a deduction may or may not be taken for the error. The standard for an individual question will state the policy.
- *Parallel solutions* – When a student presents two or more complete "solutions" to a problem, without choosing one to be graded, these are called parallel solutions. In such a case, you should read and score each solution. The student's score is then determined by truncating the average of the scores.
- *Crossed-out work* – Do not read any work that a student has crossed-out or erased.
- *Three-decimal- place rule* – Students are instructed to present answers accurate to three digits to the right of the decimal point. If those digits of the student's answer agree with the correct answer (rounded or truncated), then the student's answer is correct. For example, if the correct answer is π, then all of the following are correct: 3.141, 3.142, 3.14159, 3.1428. You should read *only* the first three digits to the right of the decimal point in the student's answer.
- *No simplification needed* – The instructions for the exam state that unless otherwise specified, an answer (numeric or algebraic) need not be simplified to be given full credit. For example, if a student's answer is $e^0 - 4 + 6$, and the correct answer is 3, the student's answer is correct.
- *Immunity from further deductions* – Some types of errors, typically errors in decimal presentation or in units, may receive only a one-point deduction in a problem. The student may repeatedly make the same type of error in a problem, but to be fair, a decision has been made to penalize the student only once.
- *We do not accept mere recipes or formulas.* – The student must apply the work to the specific problem at hand.

Terminology

- *"Our" problem* – The problem as it is written on the exam.
- *Copy error* – The student makes a minor error in copying a portion of the problem or in copying the student's own work from one line to another.
- *Re-start or false start* – The student begins the problem, but then you see work unrelated to the first work. The student may have abandoned the first attempt at a solution without crossing it out and then begun a new solution.
- *Arithmetic and algebra errors* – Errors that are non-calculus errors.
- *Eligible or eligibility requirements* – In some cases, in order for a student to gain subsequent points in a problem, the solution at a particular point must meet certain requirements.
- *Recoup* – A student may have lost a point, but a later part of the work corrects the error in some way, allowing the student to regain the lost point.
- *Bald answers* – An answer without any supporting work is called a bald answer. The allocation of credit for bald answers may vary from question to question.
- *Reversal* – A student's work may contain $b-a$ rather than $a-b$, where a and b can refer to any type of mathematical object.
- *Read with the student* – If the student has made an error, you should not quit reading the student's paper. If there are still points that may be earned, you should continue to read the student's work for full credit for the remaining points to be earned. A student may make an error at the beginning of a problem, but still earn most of the points in the problem because the student reasoned correctly from the point of the error forward.
- *0 – 2 – 0 or 0, 2, 0* – This shows you which points have been earned in a sample paper. If you look at the right side of the scoring standard, point values are listed vertically for a part of the problem. In this example, the student did not earn the first point(s), earned two points on the second portion, and then did not earn the last point(s).
- *0/3 or 3/4* – These examples show that a student has earned no points out of the three possible points in the first case, and the student has earned three points out of a possible four in the second case.

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Program

On the Role of Sign Charts in AP® Calculus Exams for Justifying Local or Absolute Extrema

David Bressoud, AP Calculus Development Committee Chair, and
Caren Diefenderfer, AP Calculus Chief Reader

Sign charts can provide a useful tool to investigate and summarize the behavior of a function. We commend their use as an investigative tool. However, the Development Committee has recommended, and the Chief Reader concurs, that sign charts, by themselves, should not be accepted as a sufficient response when a problem asks for a justification for the existence of either a local or an absolute extremum of a function at a particular point in its domain. This is a policy that will take effect with the 2005 AP Calculus Exams and Reading.

1. Local Extrema, the First Derivative Test

One way to justify that a critical point is, in fact, a local maximum or a local minimum is to use the First Derivative Test. If the first derivative changes from positive immediately to the left of the critical point to negative immediately to the right of the critical point, then there is a local maximum at the critical point. Similarly, a change in the sign of the first derivative from negative to positive guarantees that there is a local minimum at the critical point. A sign chart may contain all of the necessary information to make the conclusion that there is a local maximum or minimum, but the Development Committee and Chief Reader want to see that the student knows what it is about this information that enables the appropriate conclusion. As an example, see **1987 AB4 (a)** in the appendix.

The labeled sign chart, even with the indication that f is decreasing between -3 and 1, increasing between 1 and 3, and decreasing between 3 and 5, is not, by itself, sufficient justification. We want to see the student demonstrate a knowledge of the First Derivative Test by recognizing that there is a relative minimum at $x = 1$ because f' changes from negative to positive. The word "because," while not required, is a useful indication that the student has given a reason rather than simply assembled information. Note that it would not be sufficient justification for a relative minimum at $x = 1$ if the student said "because f changes from decreasing to increasing." This is a statement of what can be meant by a local minimum rather than an appeal to an argument based on calculus. It would be acceptable to give as justification that f is decreasing to the left of $x = 1$ because f' is negative and it is increasing to the right because f' is positive.

2. Local Extrema, the Second Derivative Test

Another way to justify that a critical point is a local maximum or minimum is to use the Second Derivative Test. Again, a sign chart for the second derivative is not enough. As an example, see **2002 Form B AB5/BC5 (a)** in the appendix. After showing that the first derivative is 0 at $x = 3$ and the second derivative is $\frac{1}{2}$, the student needs to state that f has a local minimum at $x = 3$ because the first derivative is 0 and the second derivative is positive.

3. Absolute Extrema

On a closed interval, the justification of an absolute maximum or minimum can be accomplished by identifying all critical points as well as the endpoints, evaluating the function at each of these values, and then identifying which value of x corresponds to the absolute maximum or minimum of the function. The student can also use arguments based on where the function is increasing or decreasing or the amount of change in the function to explain why certain critical or end points can be eliminated as candidates for the location of a local maximum or minimum. For example, see **2001 AB3/BC3 (c)** in the appendix.

On an open interval, the only points that need to be considered are critical points, but students must indicate that they have considered what is happening over the entire interval. For example, in **1998 AB2 (b)** in the appendix, the justification for an absolute minimum at $x = -\frac{1}{2}$ requires the observation that f' is negative for all $x < -\frac{1}{2}$ and f' is positive for all $x > -\frac{1}{2}$. It would also be a correct justification to find a value of x to the left of $-\frac{1}{2}$ for which f' is negative, a value to the right of $-\frac{1}{2}$ at which f' is positive, and then to observe that $x = -\frac{1}{2}$ is the only critical point for the function.

Appendix

1987 AB4 (a)

Let f be the function given by $f(x) = 2\ln(x^2+3) - x$ with domain $-3 \le x \le 5$. Find the x-coordinate of each relative maximum point and each relative minimum point of f. Justify your answer.

Solution

$$f'(x) = 2 \cdot \frac{2x}{x^2+3} - 1 = -\frac{(x-3)(x-1)}{x^2+3}.$$

	-3 to 1	1 to 3	3 to 5
f'	−	+	−
f	dec	inc	dec

There is a relative minimum at $x = 1$ because f' changes from negative to positive.
There is a relative maximum at $x = 3$ because f' changes from positive to negative.

Comment

The sign chart, by itself, is not sufficient justification. We need to see that the student knows what it is about the sign chart that implies a relative minimum at $x = 1$ and a relative maximum at $x = 3$.

2002 Form B AB5/BC5 (a)

Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$. Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

Solution

Since $\frac{dy}{dx} = 0$ when $x = 3$, the graph of $y = f(x)$ is tangent to the line $y = -2$ at the point $(3, -2)$. The second derivative is equal to

$$\frac{d^2y}{dx^2} = \frac{-y - y'(3-x)}{y^2}, \quad \text{and therefore } f''(3) = \frac{-(-2) - 0 \cdot (3-3)}{(-2)^2} = \frac{1}{2} > 0.$$

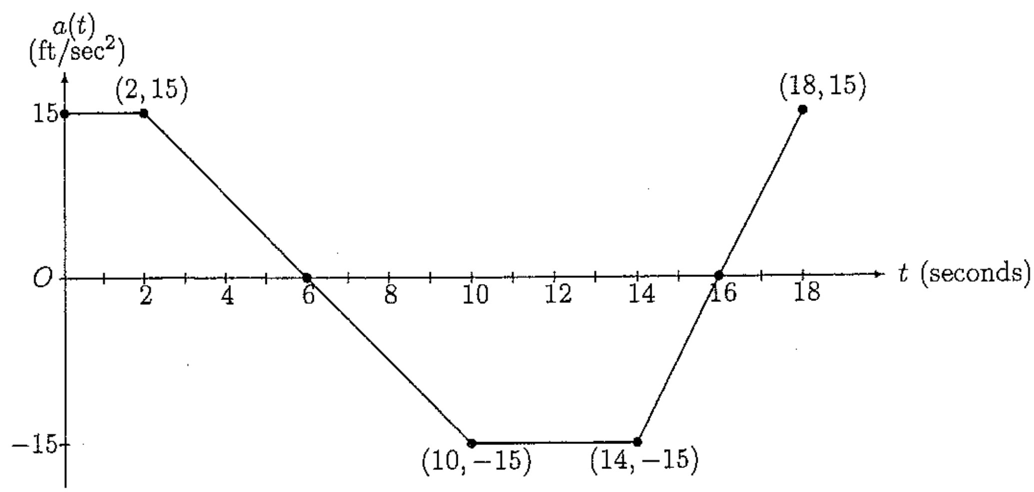
Since the first derivative is 0 and the second derivative is positive, there must be a local minimum at $x = 3$.

Comment

In most cases, a student can use the First Derivative Test to justify a local maximum or minimum, and the sign of the first derivative can be found either by inspecting the formula for the derivative, by inspecting the graph of the derivative that has been provided, or by evaluating the derivative at values on either side of the critical value. The situation in this problem is more difficult because there is no explicit representation of either the function or its derivative.

The student who tries to use the First Derivative Test to justify that there is a local minimum at $x = 3$ needs to explain why the derivative must be negative to the left of $x = 3$ and positive to the right of this value. The first step in a justification by the First Derivative Test is to observe that $y = f(x)$ is a solution of a first order differential equation for $1 < x < 5$, and so it must be continuous on that interval. The next step is to state that since $y = -2$ when $x = 3$, there must be an open interval containing 3 on which $y < 0$. On this open interval and to the left of $x = 3$ we have $x < 3$ and $y < 0$, so $\frac{dy}{dx} = \frac{3-x}{y} < 0$. On this open interval and to the right of $x = 3$ we have $x > 3$ and $y < 0$, so $\frac{dy}{dx} > 0$. We can now conclude that f has a local minimum at $x = 3$ because f' changes sign from negative to positive. This is a problem for which it is much easier to justify the answer using the Second Derivative Test.

2001 AB3/BC3 (c)



A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above. On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

Solution

Since $v'(t) = a(t)$, the derivative of v is zero only at $t = 6$ and $t = 16$. The four values that need to be checked are $t = 0$, 6, 16, and 18.

$$\begin{aligned} v(0) &= 55 \text{ ft/sec}, \\ v(6) &= 55 + \int_0^6 a(t)\,dt \;=\; 55 + 30 + 30 \;=\; 115 \text{ ft/sec}, \\ v(16) &= v(6) + \int_6^{16} a(t)\,dt \;=\; 115 - 30 - 60 - 15 = 10 \text{ ft/sec}, \\ v(18) &= v(16) + \int_{16}^{18} a(t)\,dt \;=\; 10 + 15 = 25 \text{ ft/sec}. \end{aligned}$$

The car's absolute maximum velocity is 115 ft/sec, occurring at $t = 6$.

Comment

The student can also argue from the sign of $v'(t)$ that the velocity is increasing on the intervals $[0, 6]$ and $[16, 18]$ and decreasing on the interval $[6, 16]$, and therefore the only candidates for the location of the absolute maximum are at $t = 6$ and $t = 18$. Furthermore, the student can argue that since the area between the graph of $a(t)$ and the t-axis for $6 \le t \le 16$ is greater than the area between the graph of $a(t)$ and the t-axis for $16 \le t \le 18$, the velocity at $t = 6$ must be greater than the velocity at $t = 18$, and so the absolute maximum velocity occurs at $t = 6$. For this particular problem, the student still needs to find the velocity at $t = 6$.

1998 AB2 (b)

Let f be the function given by $f(x) = 2xe^{2x}$. Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

Solution

$$\begin{aligned} f'(x) &= 2e^{2x} + 2x \cdot 2e^{2x} \;=\; 2e^{2x}(1 + 2x), \\ f'(x) &= 0 \quad \text{at} \quad x = -\frac{1}{2}. \end{aligned}$$

f'	$-$	$+$
f	dec	inc

$-\frac{1}{2}$

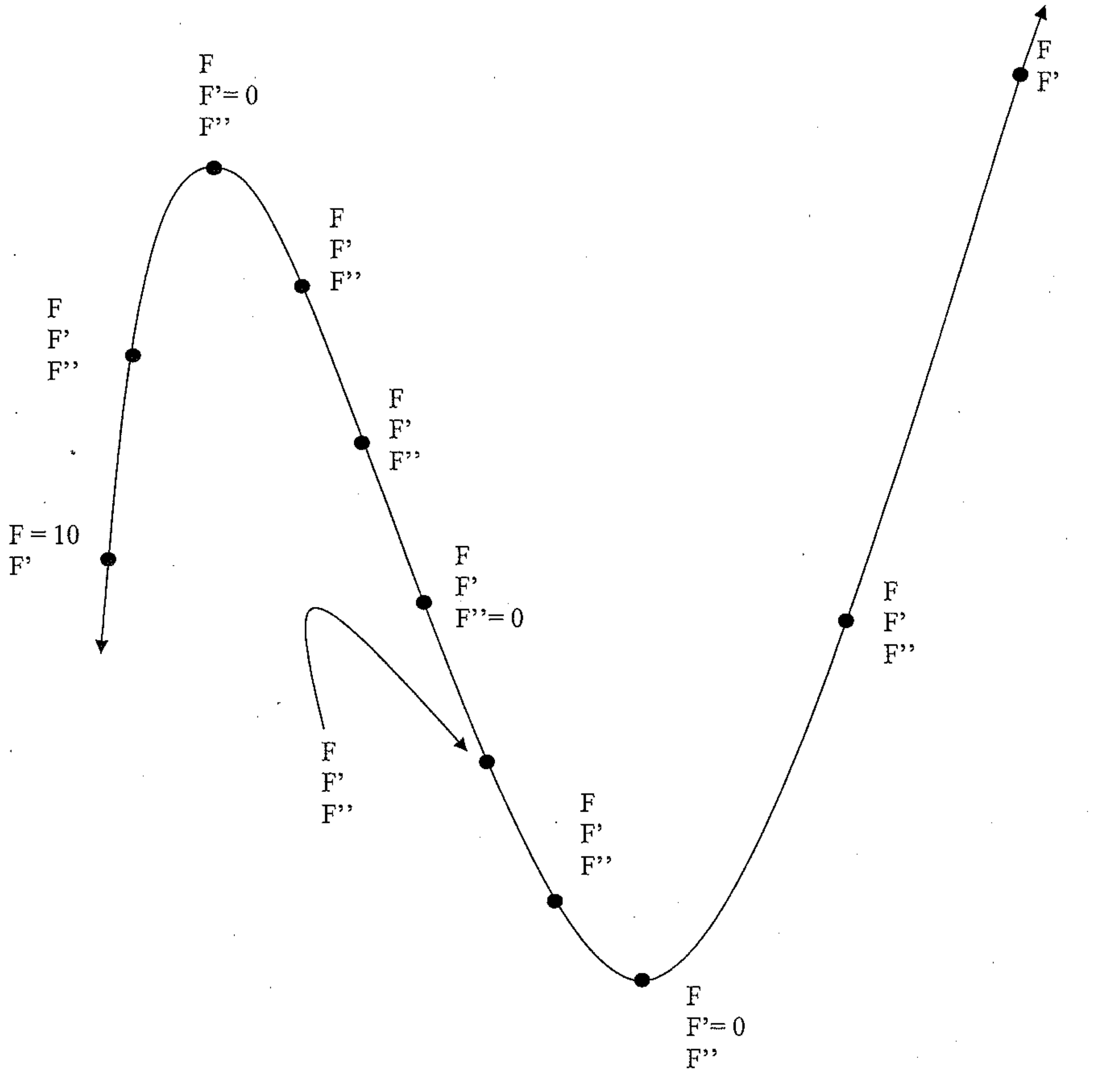
There is an absolute minimum at $x = -\frac{1}{2}$ because $f'(x) < 0$ for all $x < -\frac{1}{2}$ and $f'(x) > 0$ for all $x > -\frac{1}{2}$.

Comment

The key to justifying that we have an absolute minimum at $x = -\frac{1}{2}$ is that the derivative is negative for all $x < -\frac{1}{2}$ and positive for all $x > -\frac{1}{2}$. It is not enough to establish that the derivative changes sign from negative to positive at $x = -\frac{1}{2}$.

An equally valid justification would be that the derivative changes sign from negative to positive at $x = -\frac{1}{2}$ and $x = -\frac{1}{2}$ is the only critical point.

The "S" curve . . . determine a plausible value for F, F' and F'' for each point as indicated



BC Calculus – Unit $\ln e^4$

The secret to B^2 problems revealed . . .

So you want to solve the B^2 problems, huh? Well, you must understand (not just memorize) the concepts outlined on this document. You can do it, and it is not terribly difficult . . . spend the time and process the contents of this document. These are the questions you need to ask yourself when attacking these problems . . .

Topic: Extrema

Where should one look for absolute extrema?

Where should one look for local extrema?

What are critical points? Are they always extrema? If not, under what circumstances and provide an example.

How would you describe the difference between absolute and relative extrema? When could they be the same thing?

Given a function defined on a closed interval, outline the process you would use to find absolute extrema.

Topic: Mean Value Theorem and its Consequences

State the Mean Value Theorem for Derivatives (be precise with the details . . . this is important).

Apply the Mean Value Theorem for Derivatives if it applies; if not, justify what conditions have been violated:

$f(x) = x^2 - 4$; $[3, 6]$

$g(x) = x^{1/3}$; $[-1, 1]$

$h(x) = \dfrac{x-2}{x-1}$; $[0, 3]$

Topic: Mean Value Theorem and its Consequences (con't)

If $f' > 0$, what do we know about f?

If $f' < 0$, what do we know about f?

If f is increasing, what do we know about f'?

If f is decreasing, what do we know about f'?

How would you describe a relative maximum of f in terms of f'? Be very clear, and make sure your description distinguishes itself from a relative minimum.

Topic: Connecting f, f' and f''

What is the First Derivative Test for Local Extrema? State clearly.

Topic: Connecting f, f' and f'' (con't)

How would you define concavity?

If terms of f'', what occurs when f is concave up?

If terms of f'', what occurs when f is concave down?

What is an inflection point for the function f? Where should one look for an inflection point?

Is it possible to use f'' to find local extrema? If so, how would that work? Does it ever ***not*** work? If so, when?

If terms of f', what occurs when f is concave up?

If terms of f', what occurs when f is concave up?

When f is positive, what do we know, if anything, about f' and f''?

Topic: Connecting f, f' and f'' (con't)

When f is negative, what do we know, if anything, about f' and f''?

When f is increasing, what do we know, if anything, about f' and f''?

When f is decreasing, what do we know, if anything, about f' and f''?

When f' is positive, what do we know, if anything, about f and f''?

When f' is negative, what do we know, if anything, about f and f''?

When f' is increasing, what do we know, if anything, about f and f''?

When f' is decreasing, what do we know, if anything, about f and f''?

When f'' is positive, what do we know, if anything, about f and f'?

When f'' is negative, what do we know, if anything, about f and f'?

Topic: Sketching f, f' and f''

<u>Sketch the curves:</u>

Increasing at an increasing rate

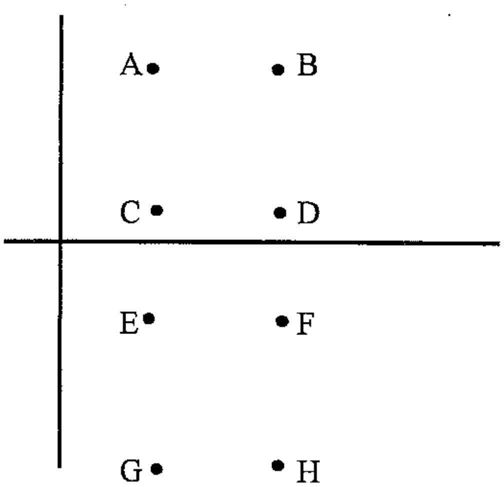
Increasing at a decreasing rate

Decreasing at an increasing rate

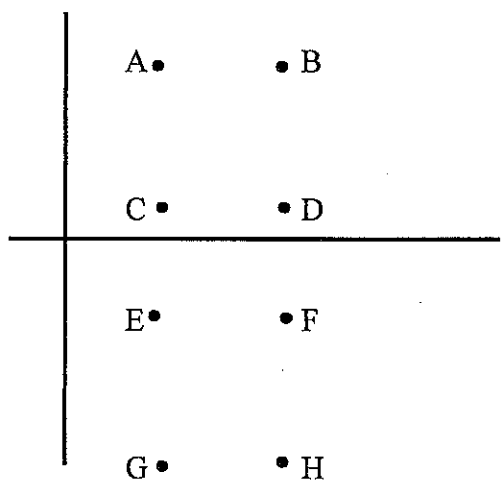
Decreasing at a decreasing rate

Based upon the given conditions on the chart, sketch a curve between any two points on the provided grid

f	f'	f''
<0	<0	<0

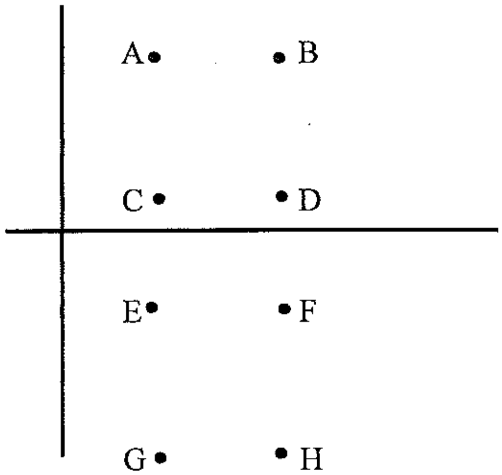


f	f'	f''
>0	>0	>0

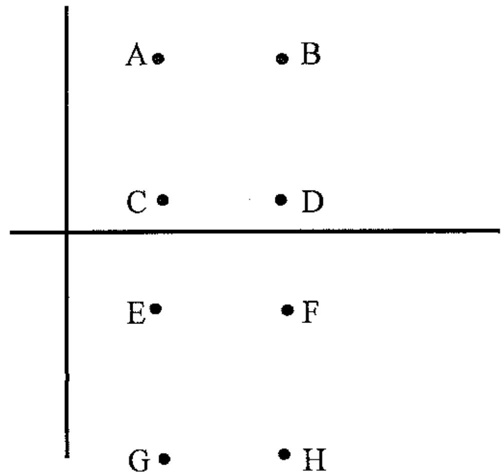


Topic: Sketching f, f' and f''(con't)

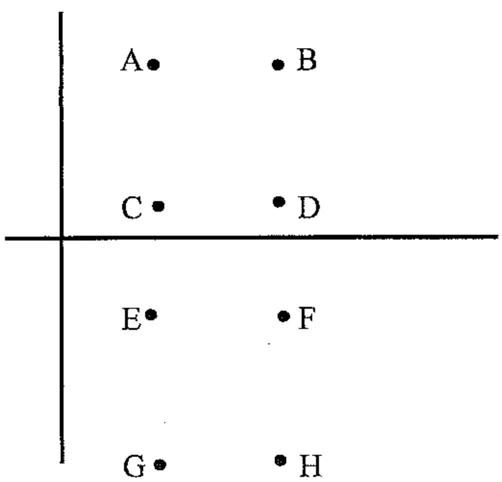
f	f'	f''
>0	>0	<0



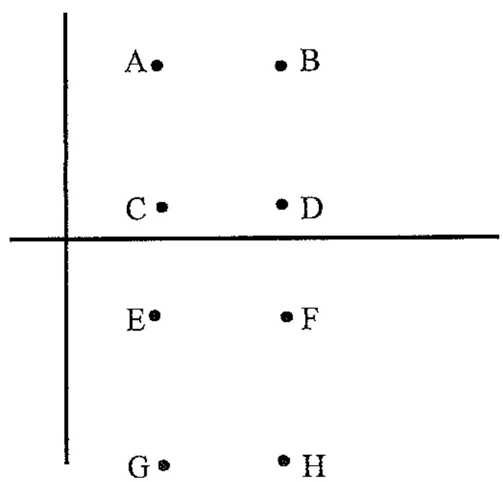
f	f'	f''
<0	>0	>0



f	f'	f''
>0	<0	<0

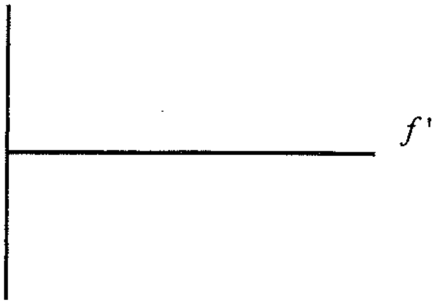


f	f'	f''
<0	<0	>0

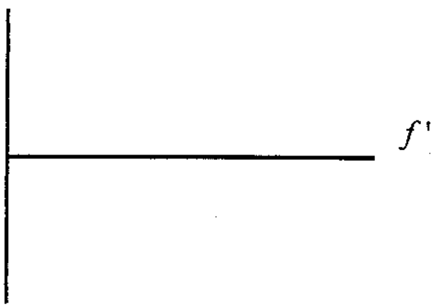


Topic: Sketching f, f' and f''(con't)

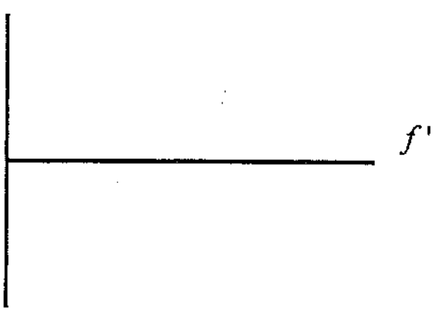
If f is increasing at an increasing rate, how do you sketch f'?



If f is increasing at a decreasing rate, how do you sketch f'?



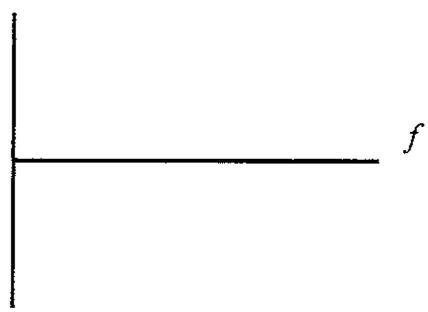
If f is decreasing at an increasing rate, how do you sketch f'?



If f is decreasing at a decreasing rate, how do you sketch f'?

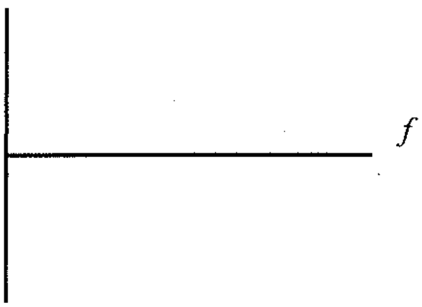
If f' is below the x-axis and going away from the x-axis, how is f sketched?

If f' is above the x-axis and going away from the x-axis, how is f sketched?

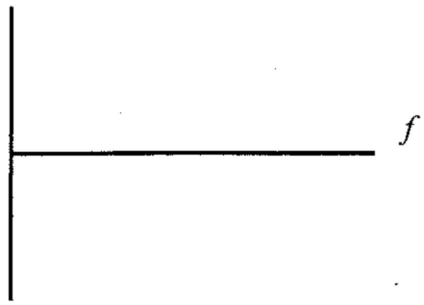


Topic: Sketching f, f' and f''(con't)

If f' is below the x-axis and going toward the x-axis, how is f sketched?

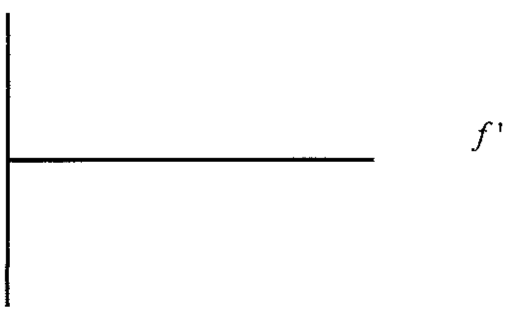


If f' is above the x-axis and going toward the x-axis, how is f sketched?

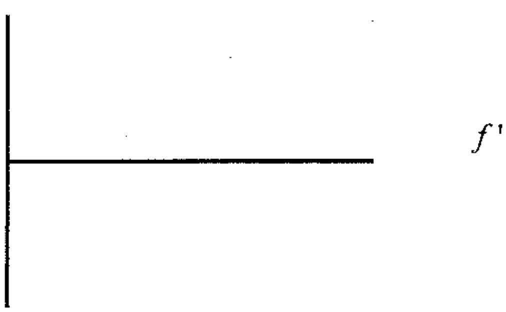


What places on the f' graph correspond to extrema for f? How do you know if that place represents a local maximum or a minimum? Sketch an example of a f' graph that indicates a local maximum. Also, sketch an example of a f' graph that indicates a local minimum. Specify the distinct characteristic(s) of f' that illustrates the distinction between the extrema.

Local maximum



Local minimum



When we now incorporate f'' into the mix, we have to be careful. It will be very common for me to give to you the graph of f' and have you sketch both f and f''. Remember: going from f' to f'' is basically the same as going from f to f' – see f' in terms of the four curves, then sketch above or below (the x-axis), away or toward (the x-axis).

Analytic Problems using f, f' and f''

You will be given a function, and asked to find the following things:

- Σ Critical points
- Σ Extrema
- Σ Intervals where f is increasing and/or decreasing
- Σ Points of inflection
- Σ Intervals where f is concave up and/or concave down

Write a step-by-step process for each of the bullets above; pay special attention to details and notation.

Critical points:

Relative extrema:

Intervals where f is increasing and/or decreasing:

Points of inflection:

Intervals where f is concave up and/or concave down: