

**Arithmetic Sequence – the difference between consecutive terms is a constant ( $d$ )  
 $d$  is called the common difference.**

Identifying an Arithmetic Sequence:

ex: 12, 16, 20, 24, ...

$$16 - 12 = 4$$

yes, the sequence is arithmetic  $d = 4$

$$20 - 16 = 4$$

$$24 - 20 = 4$$

ex: 87, 80, 73, 66, ...

ex: 1, 3, 6, 10, ...

### Arithmetic Sequence Formulas

Recursive

$$\begin{cases} a_1 = a \text{ given value} \\ a_n = a_{n-1} + d \end{cases}$$

Explicit

$$a_n = a_1 + (n - 1)d$$

$a_1 = \text{first term}$

$a_n = \text{nth term}$

$a_{n-1} = \text{the previous term}$

$d = \text{common difference}$

Find the first five terms of the arithmetic sequence:

ex:  $\begin{cases} a_1 = 20 \\ a_n = a_{n-1} + 8 \end{cases}$        $a_1 = 20$

$a_2 = 20 + 8 = 28$

Recursive formula       $a_3 = 28 + 8 = 36$

$a_4 = 36 + 8 = 44$

$a_5 = 44 + 8 = 52$       20, 28, 36, 44, 52 ...

ex:  $a_n = 16 + (n - 1)3$        $a_1 = 16 + (1 - 1)3 = 16$

$a_2 = 16 + (2 - 1)3 = 19$

Explicit formula       $a_3 = 16 + (3 - 1)3 = 22$

$a_4 = 16 + (4 - 1)3 = 25$

$a_5 = 16 + (5 - 1)3 = 28$       16, 19, 22, 25, 28, ...

Find the 20<sup>th</sup> term of each of the above sequences:

20, 28, 36, 44, 52 ...

$$\begin{cases} a_1 = 20 \\ a_n = a_{n-1} + 8 \end{cases}$$

16, 19, 22, 25, 28, ...

$$a_n = 16 + (n - 1)3$$

Example: Find the 75<sup>th</sup> term of the arithmetic sequence: 12, 4, -4, -12, ...

APPLICATION: Suppose you have already saved \$75 toward the purchase of a new phone. You plan to save \$12 each week from your part-time job. What is the amount you will have saved after 26 weeks?

<u>Week</u>	<u>Savings</u>	$a_n = a_1 + (n - 1)d$
1	\$87	
2	\$99	$a_n = 87 + (n - 1)12$
3	\$111	
4	\$123	$a_{26} = 87 + (26 - 1)12 = \$387$

**Arithmetic Mean** – the arithmetic mean of any two numbers is the average of the two numbers.

$\text{Arithmetic Mean} = \frac{\text{sum of the two numbers}}{2}$
--

We can use the arithmetic mean to find a missing term from an arithmetic sequence.

ex: Find the missing term of the arithmetic sequence

..., 84, \_\_\_\_, 110, ...

$$\text{arithmetic mean} = \frac{84+110}{2} = 97$$

..., 84, 97, 110, ...

ex: Find the arithmetic mean  $a_n$  of the given terms.

$$a_{n-1} = 0.3$$

$$a_{n+1} = 1.9$$

ex: find the missing terms of the arithmetic sequence.

..., 12, \_\_\_\_, \_\_\_\_, \_\_\_\_, 28, ...