Algebra 2 Arithmetic Series Notes Date:

Series – the sum of the terms of a sequence

Finite sequence (has a last term)

$6, 9, 12, 15, 18$

Finite series

$6 + 9 + 12 + 15 + 18$

Infinite sequence (doesn't have a last term)

$3, 7, 11, 15, 19, \ldots$

Infinite series

$3 + 7 + 11 + 15 + 19 + \cdots$

Arithmetic Series – a series whose terms from an arithmetic sequence

ex: $5, 9, 13, 17, 21, 25, 29$ finite arithmetic sequence

$5 + 9 + 13 + 17 + 21 + 25 + 29$ finite arithmetic series

Evaluate the series: $5 + 9 + 13 + 17 + 21 + 25 + 29 = 119$

Sum of a Finite Arithmetic Series: $a_1 + a_2 + a_3 + \cdots + a_n$

$$\boxed{S_n = \frac{n}{2}(a_1 + a_n)}$$

$n = number\ of\ terms$

$a_1 = first\ term$

$a_n = nth\ term\ (last\ term)$

ex: use the formula to evaluate the sum of the arithmetic series: $5 + 9 + 13 + 17 + 21 + 25 + 29$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_7 = \frac{7}{2}(5 + 29)$$

$$S_7 = 3.5(34) = 119$$

ex: use the formula to evaluate the sum of the arithmetic series:

$12 + 15 + 18 + 21 + 24 + 27 + 30 + 33$

ex: use the formula to evaluate the sum of the arithmetic series:

$100 + 88 + 76 + 64 + 52$

Evaluate the series to the given term.

$-5 - 25 - 45 - 65 - \cdots$ to the 9th term

arithmetic $a_1 = -5$

$d = -20$

$a_n = a_1 + (n-1)d$

$a_9 = -5 + (9-1)(-20)$

$a_9 = -165$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_9 = \frac{9}{2}(-5 + -165)$$

$$S_9 = 4.5(-170) = -765$$

Evaluate the series to the given term.

$8 + 18 + 28 + 38 + \cdots$ to the 12th term

Summation Notation

$$\sum_{n=1}^{3}(5n+1)$$

$$\sum_{n=1}^{3}(5n+1) = (5\cdot 1+1)+(5\cdot 2+1)+(5\cdot 3+1)$$
$$= \quad 6 \quad + \quad 11 \quad + \quad 16 \quad = 33$$

or using the formula: $S_n = \frac{n}{2}(a_1 + a_n)$

$S_3 = \frac{3}{2}(6+16)$

$S_3 = 1.5(22) = 33$

Ex: Use summation notation to write the series for the first 50 terms: $8 + 14 + 20 + \cdots$

$8 + 14 + 20 + \cdots$

Arithmetic

$a_1 = 8$

$d = 6$

find the explicit formula: $a_n = a_1 + (n-1)d$

$a_n = 8 + (n-1)6$

$a_n = 8 + 6n - 6$

$a_n = 6n + 2$

$$\sum_{n=1}^{50}(6n+2) =$$

Number of Terms = 50

First Term: $6\cdot 1 + 2 = 8$

Last Term: $6\cdot 50 + 2 = 302$

$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{50} = \frac{50}{2}(8+302)$

$S_n = 25(310) = 7750$

Evaluating an Arithmetic Series (using Summation Notation)

$$\sum_{n=1}^{20} (-2n + 3)$$

a) number of terms = 20

b) first term $= -2(1) + 3 = 1$
last term $= -2(20) + 3 = -37$

c) evaluate the series: $S_{20} = \frac{20}{2}(1 + -37)$
$S_{20} = 10(-36) = -360$

ex:

$$\sum_{x=5}^{8} (7x - 10)$$